

**RYERSON UNIVERSITY**  
**MTH 714 LAB#8**  
**DAY: OCTOBER 30, 2008**

1. For the following formula, either prove that it is valid or give a falsifying interpretation

$$\forall x \forall y \forall z [p(x, x) \wedge (p(x, z) \rightarrow (p(x, y) \vee p(y, z)))] \rightarrow \exists y \forall z p(y, z).$$

2. Show that

$$\forall x (p(x) \vee q(x)) \rightarrow (\forall x p(x) \vee \forall x q(x))$$

is not valid using the semantic tableaux method.

3. Prove, in the Hilbert proof system for predicate logic, that

$$\vdash \exists x (p(x) \rightarrow q(x)) \rightarrow (\forall x p(x) \rightarrow \exists x q(x)).$$

4. What is wrong with the following “proof” of

$$\{\exists x A(x), \exists x B(x)\} \vdash \exists x (A(x) \wedge B(x))?$$

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|----|--|----------------------|
| 1. | $\{\exists x A(x), \exists x B(x)\} \vdash \exists x A(x)$               | Assumption           |
| 2. | $\{\exists x A(x), \exists x B(x)\} \vdash A(c)$                         | C-Rule 1             |
| 3. | $\{\exists x A(x), \exists x B(x)\} \vdash \exists x B(x)$               | Assumption           |
| 4. | $\{\exists x A(x), \exists x B(x)\} \vdash B(c)$                         | C-Rule 3             |
| 5. | $\{\exists x A(x), \exists x B(x)\} \vdash A(c) \wedge B(c)$             | Conjunction Rule 2,4 |
| 6. | $\{\exists x A(x), \exists x B(x)\} \vdash \exists x (A(x) \wedge B(x))$ | Existential Rule 5   |