RYERSON UNIVERSITY
MTH 714 LAB#7
DAY: OCTOBER 23, 2008

1. Consider the following formulas A_1, A_2, A_3, which express that the predicate is reflexive, symmetric, and transitive:

$$A_1 = \forall x p(x, x)$$
$$A_2 = \forall x \forall y (p(x, y) \rightarrow p(y, x))$$
$$A_3 = \forall x \forall y \forall z (p(x, y) \wedge p(y, z) \rightarrow p(x, z))$$

   Show that none of the three formulas is a consequence of the other two by presenting structures which are models for two of the formulas but not the third.

2. Which of the following structures are models for the formula

$$F = \exists x \exists y \exists z (p(x, y) \wedge p(z, y) \wedge p(x, z) \wedge \neg p(z, x))?$$

   (a) $I_1 = (\mathbb{N}, \{(m, n) | m, n \in \mathbb{N}, m < n\})$

   (b) $I_2 = (\mathbb{N}, \{(m, m + 1) | m \in \mathbb{N}\})$

   (c) $I_3 = (\mathcal{P}(\mathbb{N}), \{\subseteq\})$

3. Find a closed satisfiable formula F such that for every model I of F, the domain of I must have at least three elements.

4. For the following formula, either prove that it is valid or give a falsifying interpretation

$$\forall x \forall y \forall z [p(x, x) \wedge (p(x, z) \rightarrow (p(x, y) \vee p(y, z)))] \rightarrow \exists y \forall z p(y, z).$$