RYERSON UNIVERSITY
MTH 714 LAB#2
DAY: SEPTEMBER 18, 2008

1. Determine whether the following formulas are valid (tautologies) or not:

   (a) $((p \to q) \to q) \to q$

   (b) $((p \to q) \to p) \to p$

   (c) $(p \wedge q) \to (p \vee r)$

   (d) $(p \vee \neg(q \wedge r)) \to ((p \leftrightarrow r) \vee q)$

2. Determine whether the following pairs of formulas are equivalent:

   (a) $(p \to q) \to p$ and p

   (b) $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$

   (c) $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$

   (d) $p \vee (q \leftrightarrow r)$ and $(p \vee q) \leftrightarrow (p \vee r)$

   (e) $(p \vee (q \vee r)) \wedge (r \vee \neg p)$ and $(q \wedge \neg p) \vee r$

3. (*Duality Theorem*) (a) If A is a formula involving only $\neg$, $\wedge$ and $\vee$ as its connectives, and A' results from A by replacing each $\wedge$ by $\vee$ and each $\vee$ by $\wedge$, show that A is valid if and only if $\neg A'$ is valid. [Hint: Use structural induction.]

   (b) Also, show that if $A \to B$ is valid, for some formula B which only uses $\neg$, $\vee$ and $\wedge$, then $B' \to A'$ is also valid.

4. (a) Show that, if A is a formula containing $\to$ and $\vee$ as its only connectives, then $v(A) = T$ for every assignment v which assigns T to every atom.

   (b) Deduce that the set of connectives $\{\to, \vee\}$ cannot generate all Boolean operators.

5. Show that every one of the connectives from the set $\{\wedge, \to, \leftrightarrow\}$ can be expressed in terms of the other two.

6. Prove the following theorem: If $\{\circ\}$ is an adequate set of connectives in propositional logic, where $\circ$ is a binary operator, then either $\circ = \uparrow$ or $\circ = \downarrow$.