RYERSON UNIVERSITY
MTH 714 LAB#1 - SOLUTIONS
DAY: SEPTEMBER 11, 2008

1.

p	q	$\neg(p \leftrightarrow \neg(q \wedge \neg p))$
T	T	F
T	F	F
F	T	F
F	F	T

2. (a) All assignments such that $v(p) = v(q) =$T as well as the assignment $v(p) =$T, $v(q) = v(r) =$F.

   (b) The only such assignment is $v(p) = v(q) =$T.

3. Notice the following property of the $\leftrightarrow$ connective:

A	B	$A \leftrightarrow B$
T	T	T
T	F	F
F	T	F
F	F	T

   Changing the value of either formula A or B, while keeping the value of the other formula same, will change the value of $A \leftrightarrow B$.

   Now, an example of a formula in atoms p, q, and r having the required property is

$$(p \leftrightarrow q) \leftrightarrow r$$

4. We will prove this using the structural induction on A.

   Suppose $A = \neg B$, for some formula B which satisfies the induction hypothesis; i.e. the truth table for B has an even number of rows in which B is true as those in which B is false. Then, the truth table for A will have the same property since $\neg$ will simply invert the truth values for B. Therefore, A has the required property.

   Now, suppose $A = B \leftrightarrow C$, where B and C satisfy the inductive hypothesis. By inductive hypothesis, the truth tables for B and C both have an even number of rows in which the formulas are true and an even number of rows in which they are false. So, assume m is an even number which is the number of rows in which B is true, and k plays the same role for C. We may also assume that both B and C use the same number of atoms so both formulas have truth tables with 2^n rows.

   Consider the truth table for A:

B	C	$A = B \leftrightarrow C$
T	T	T
T	F	F
F	T	F
F	F	T

Now, suppose there are i rows in this truth table where $v(B) = v(C) = T$, j rows in which $v(B) = T, v(C) = F$, r rows in which $v(B) = F, v(C) = T$, and s rows in which $v(B) = v(C) = F$.

Then:

$$\begin{aligned} i + j &= m \\ r + s &= 2^n - m \\ i + r &= k \\ j + s &= 2^n - k \end{aligned}$$

We also have:

$$i + j + r + s = 2^n$$

So, since k and m are even numbers (by inductive hypothesis), by subtracting and adding pairs of these five equalities, one can show that the sum $i + s$ is an even number. One way to show this is e.g. as follows:

$$2i + j + r = m + k$$

is even, and so is:

$$i - s = (2i + j + r) - (i + j + r + s) = m + k - 2^n$$

which implies that

$$i + s = (i - s) + 2s = m + k - 2^n + 2s$$

is an even number, too.

But, that is precisely the number of the rows for $A = B \leftrightarrow C$ in which $v(A) = T$. Therefore, we proved that every formula built from atoms and connectives $\neg$ and $\leftrightarrow$, and which is not an atom itself, has the required property.