RYERSON UNIVERSITY
MTH 714
ASSIGNMENT #1 (due:October 14, 2008)

1. Construct a propositional formula A such that the formula

$$(((A \wedge q) \rightarrow \neg p) \rightarrow ((p \rightarrow \neg q) \rightarrow A))$$

is valid.

2. Show that the set of operators

$$\{\leftrightarrow, \vee, \mathit{false}\}$$

is adequate, where *false* is a fixed contradictory formula (i.e. a constant F function on $\{T, F\}$).

3. Prove that

$$\vdash ((\neg B \rightarrow \neg A) \rightarrow A) \rightarrow A$$

(a) using the semantic tableaux method;

(b) using the Gentzen axiom system $\mathcal{G}$

4. (a) Show that

$$\vdash_{\mathcal{H}} (\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$$

(b) Show that, if we replace Axiom 3 of the Hilbert proof system $\mathcal{H}$ with

$$(\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B),$$

we can derive Axiom 3 as a theorem in this new proof system.

(c) Using this, explain why we can replace the third axiom of the Hilbert proof system with

$$(\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$$

without changing the completeness; i.e. why we are still able to prove all valid formulas as theorems.

5. Using resolution, prove the following:

   (a) the set of clauses

   $$F = \{pq\bar{r}, \bar{p}, pqr, p\bar{q}\}$$

   is unsatisfiable.

   (b) the formula

   $$A = (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg r) \vee (q \wedge r) \vee p$$

   is valid.