CONJECTURES ON COPS AND ROBBERS

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Abstract. We consider some of the most important conjectures in the study of the game of Cops and Robbers and the cop number of a graph. The conjectures touch on diverse areas such as algorithmic, topological, and structural graph theory.

1. Introduction

The game of Cops and Robbers and its associated graph parameter, the cop number, have been studied for decades but are only now beginning to resonate more widely with graph theorists. One of the reasons for this owes itself to a challenging conjecture attributed to Henri Meyniel. Meyniel's conjecture, as it is now called, is arguably one of the deepest in the topic, and will likely require new techniques to tackle. The conjecture has attracted the attention of the graph theory community, and has helped revitalize the topic of Cops and Robbers. See Section 2 below.

As the game is not universally known, we define it here and provide some notation (it is customary to always begin a Cops and Robbers paper with the definition of cop number; regardless of best intentions, it is difficult to buck the trend). We consider only finite, undirected

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graphs in this paper, although we can play Cops and Robbers on infinite graphs or directed graphs in the natural way. Further, since the cop number is additive on connected components, we consider only *connected graphs*.

We now formally define the game. Cops and Robbers is a game of perfect information; that is, each player is aware of all the moves of the other player. There are two players, with one player controlling a set of *cops*, and the second controlling a single *robber*. The game is played over a sequence of discrete time-steps; a *round* of the game is a move by the cops together with the subsequent move by the robber. The cops and robber occupy vertices, and when a player is ready to move in a round they must move to a neighboring vertex. The cops move first, followed by the robber; thereafter, the players move on alternate steps. Players can *pass*, or remain on their own vertices. Observe that any subset of cops may move in a given round. The cops win if after some finite number of rounds, one of them can occupy the same vertex as the robber. This is called a *capture*. The robber wins if he can evade capture indefinitely. Note that the initial placement of the cops will not affect the outcome of the game, as the cops can expend finitely many moves to occupy a particular initial placement (the initial placement of the cops may, however, affect the length of the game).

Note that if a cop is placed at each vertex, then the cops are guaranteed to win. Therefore, the minimum number of cops required to win in a graph G is a well-defined positive integer, named the *cop number* of the graph G. The notation $c(G)$ is used for the cop number of a graph G. If $c(G) = k$, then G is *k-cop-win*. In the special case $k = 1$, G is *cop-win*.

For a familiar example, the cop number of the Petersen graph is 3. In a graph G, a set of vertices S is *dominating* if every vertex of G not in S is adjacent to some vertex in S. The *domination number* of a graph G is the minimum cardinality of a dominating set in G. Note that 3 cops are sufficient in the Petersen graph, as the domination number upper bounds the cop number. See Figure 1. This bound, however, is far from tight. For example, paths (or more generally, trees), have cop number 1.

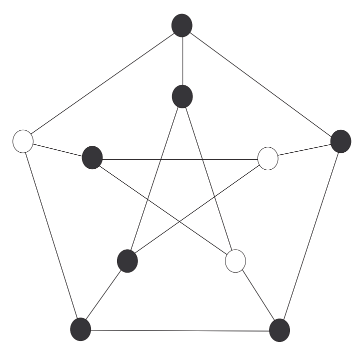

FIGURE 1. The Petersen graph with white vertices dominating.

There are now a number of conjectures that have arisen on Cops and Robbers, touching on many areas including algorithmic, topological, and structural graph theory. Some of these are more or less known. We will discuss these in the sections below.

When I am discussing Cops and Robbers with a newcomer, I am aware of the following, purely tongue-in-cheek principle:

Cops and Robbers Principle: Once you learn about Cops and Robbers you are compelled to prove results about it.

The Cops and Robbers Principle, while itself is unverifiable, does seem fairly pervasive. One reason for this owes to the fact that the cop number, first defined in 1984, remains an unfamiliar parameter to many graph theorists. The cop number has limited connections (at least based on our current knowledge) to commonly studied graph parameters; this makes the field both challenging and fresh. The game is also simple to define and easy to play. You can even play it with some coins on a drawn graph with non-mathematicians.

Another reason why the Principle so often holds owes to the wealth of variations possible with the game. Almost every talk I give on the subject at a conference inspires the audience to spawn at least one (occasionally new) variation. This is not surprising as mathematicians have active imaginations, and Cops and Robbers definitely provides a fertile playground for the imaginative. One of my early mentors, the late lattice theorist Gunter Bruns, told the story of how he knew a mathematician who quit the field to become a poet. His reason for quitting was that he did not have enough imagination!

As a concrete example of this aspect of the Principle, at the 2014 SIAM Conference on Discrete Mathematics held in Minneapolis, colleagues Shannon Fitzpatrick and Margaret-Ellen Messenger suggested the new variant *Zombies and Survivors*. In this game, the zombies (cops) have minimal intelligence, and always move directly towards the survivor (robber) along a shortest path (if there is more than one such path, then the zombies get to choose which one). We laughed at the following instance of the game. Consider a group of $\lfloor n/2 \rfloor - 2$ zombies on a cycle C_n, where $n \geq 4$. Place them on distinct, consecutive vertices, so they form a path of zombies. The survivor then chooses a vertex distance two from the "lead" zombie (that is, the leaf of the zombie path). This placement of the zombies would result in zombies endlessly chasing the survivor in an orderly path. The survivor is forever just out of reach of the massive horde of hungry zombies! After learning about this variant, I told my colleagues they maybe watching too many horror movies. In all seriousness, this variant speaks volumes about the broad appeal of the game.

The historical origin of the game is an interesting story in its own right. The game of Cops and Robbers was first considered by Quilliot [38] in his doctoral thesis. The game remained largely unknown at this time until it was considered independently by Nowakowski and Winkler [34]. According to Google Scholar, that 5 page paper is the most cited of either author! Mathematics is no exception to the slogan "less is more."

Interestingly, both [34, 38] consider the game played with only one cop. In particular, they both focus on characterizing the cop-win graphs. The introduction of the cop number came a year later in 1984 with the important work of Aigner and Fromme [1].

Our book summarizes much of the research on Cops and Robbers up to 2011; see [17]. The interested reader is referred there for a broader background than provided here; see also the surveys [2, 8, 9, 27].

2. Meyniel's conjecture

Graphs with cop number larger than one are not particulary well understood. The cop-win case is, on the other hand, well characterized as we describe next.

The *closed neighborhood* of a vertex x, written $N[x]$, is the set of vertices adjacent to x (including x itself). A vertex u is a *corner* if there is some vertex v such that $N[u] \subseteq N[v]$.

A graph is *dismantlable* if some sequence of deleting corners results in the graph K_1. For example, each tree is dismantlable: delete leaves repeatedly until a single vertex remains. The same approach holds with chordal graphs, which always contain at least two simplicial vertices (that is, vertices whose neighbor sets are cliques). The following result characterizes cop-win graphs.

Theorem 1 ([34])**.** *A graph is cop-win if and only if it is dismantlable.*

The theorem provides a recursive structure to cop-win graphs, made explicit in the following sense. Observe that a graph is dismantlable if the vertices can be labeled by positive integers $\{1, 2, \ldots, n\}$, in such a way that for each $i < n$, the vertex i is a corner in the subgraph induced by $\{i, i+1, \ldots, n\}$. This ordering of $V(G)$ is called a *cop-win ordering* (in the context of chordal graph theory, this is called an *elimination ordering*). See Figure 2 for a graph with vertices labeled by a cop-win ordering.

How big can the cop number be? First notice that for every positive integer n, there is a graph with cop number n. Hypercubes, written

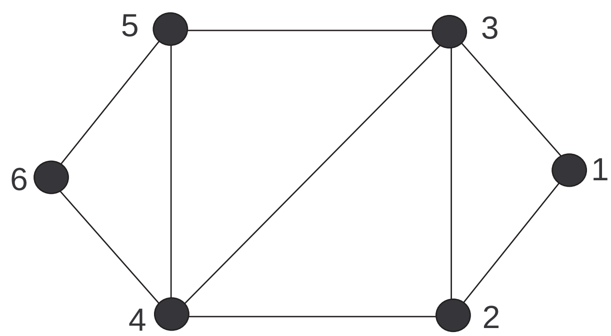

FIGURE 2. A cop-win ordering of a cop-win graph.

Q_n (where n is a non-negative integer), are a family of graphs realizing every possible cop number (if we take take Q_2 to be K_1). To see this, note that it was shown in [32] that for the hypercube Q_n of dimension n, $c(Q_n) = \lceil \frac{n+1}{2} \rceil$.

For a positive integer n, let $c(n)$ be the maximum cop number of a graph of order n (recall that we only consider connected graphs). *Meyniel's conjecture* states that there is a constant $d > 0$ such that for all positive integers n we have that

$$c(n) \leq d\sqrt{n}.$$

The conjecture was mentioned briefly in Frankl's paper [23] as a personal communication to him by Henri Meyniel in 1985 (see page 301 of [23] and reference [8] in that paper). As Meyniel has since passed away, we may never know his original motivation for the conjecture. Meyniel actually only published one short paper on Cops and Robbers, on a topic unrelated to the conjecture; see [32].

Meyniel's conjecture seemed to be largely unnoticed until recently. I may have been partly responsible for Meyniel's conjecture's rehabilitation. In 2006, I attended a small workshop organized by Geňa Hahn at the Bellairs Research Institute in Barbados, and I spoke about recent research on the cop number. The workshop was delightful, in no small part owing to the beautiful location. Jan Kratochvíl was there, and it appeared that the Cops and Robbers Principle was still in effect. He subsequently told Béla Bollobás about the parameter and conjecture, who then produced [7] (I am making this assumption based on the acknowledgement to Kratochvíl in that paper). Since then the interest in the conjecture has steadily grown. I also spoke at Bellairs about the capture time of a graph, which led to joint with Kratochvíl and others [14]. A play of the game with $c(G)$ cops is *optimal* if its length is the minimum over all possible plays for the cops, assuming the robber is trying to evade capture for as long as possible. There may be many optimal plays possible (for example, on the path P_4 with four vertices, the cop may start on either of the two vertices in the centre), but the length of an optimal game is an invariant of G. When $c(G)$ cops play on a graph G, we denote this invariant by $\mathrm{capt}(G)$ and refer to this as the *capture time* of G. In [14], the authors proved that if G is cop-win (that is, has cop number 1) of order $n \geq 5$, then $\mathrm{capt}(G) \leq n-3$. By considering small-order cop-win graphs, the bound was improved to $\mathrm{capt}(G) \leq n-4$ for $n \geq 7$ in [25]. Examples were given of planar cop-win graphs in both [14, 25] which prove that the bound of $n-4$ is optimal. In addition to these works, capture time was studied in grids [33] and hypercubes [16].

For many years, the best known upper bound for general graphs was the one proved by Frankl [23].

Theorem 2 ([23]). *If n is a positive integer, then*

$$c(n) = O\left(n\frac{\log\log n}{\log n}\right).$$

I spoke about the cop number at the University of Waterloo in October 2007, to a group consisting mainly of theoretical computer scientists. My talk spurred a bright doctoral student Ehsan Chiniforooshan to consider improving on known upper bounds on the cop number. The Cops and Robbers Principle was again in full force that day! Chiniforooshan exploited similar ideas with retracts and proved the following bound, giving a modest improvement to Frankl's bound.

Theorem 3 ([18]). *If n is a positive integer, then*

$$c(n) = O\left(\frac{n}{\log n}\right).$$

At the time of writing this chapter in January 2015, the conjecture is still open. The best known upper bound was proved independently by three sets of authors. Interestingly, all of them use the probabilistic method in their proofs.

Theorem 4 ([24, 31, 41]). *If n is a positive integer, then*

$$c(n) = O\left(\frac{n}{2^{(1-o(1))\sqrt{\log n}}}\right).$$

To put Theorem 4 into perspective, even proving $c(n) = O(n^{1-\epsilon})$ for any given $\epsilon > 0$ remains open.

Prałat and Wormald in some recent work proved the conjecture for random graphs [36] and for random regular graphs [37], which gives us more evidence that the conjecture is true. I tend to believe the conjecture is true on good days; when I am in a bad mood I imagine the universe contains some strange graph with cop number of larger order than $\sqrt{n}$.

There are graphs whose cop number is $\Theta(\sqrt{n})$; for example, consider the incidence graphs of finite projective planes. These graphs are of order $2(q^2 + q + 1)$, where q is a prime power, and have cop number $q + 1$. See Figure 3 for an example. The Cops and Robbers Principle was in effect when I described this graph family to the design theorist Andrea Burgess, which lead to several other families with conjectured largest cop number; see [10] .

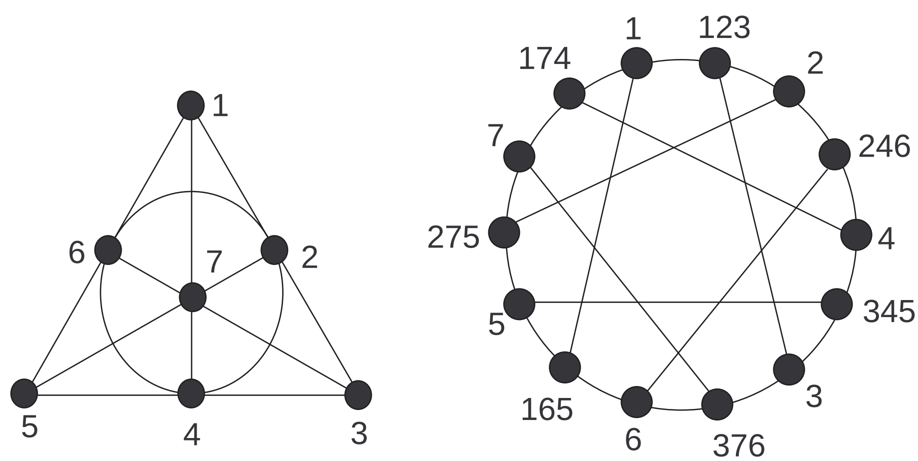

FIGURE 3. The Fano plane and its incidence graph, the Heawood graph.

Define m_k to be the minimum order of a connected graph G satisfying $c(G) \geq k$. Trivially, $m(1) = 1$ and $m(2) = 4$. The recent work [3, 4] establishes the fact that $m_3 = 10$. The unique isomorphism type of graph of order 10 with cop number 3 is the Petersen graph. It is easy to see that Meyniel's conjecture is equivalent to the property that

$$m_k = \Omega(k^2).$$

It might be fruitful to consider, therefore, the minimum orders of graph with a given cop number. We do not even know the exact value of m_4. The Petersen graph is the unique 3-regular graph of girth 5 of minimal order. A (k, g)-*cage* is a k-regular graph with girth g of minimal order. See [21] for a survey of cages. The Petersen graph is the unique $(3, 5)$-cage, and in general, cages exist for any pair $k \geq 2$ and $g \geq 3$. Aigner and Fromme [1] proved that graphs with girth 5, and degree k have cop number at least k; in particular, if G is a $(k, 5)$-cage, then $c(G) \geq k$. Let $n(k, g)$ denote the order of a (k, g)-cage. Is it true that a $(k, 5)$-cage is k-cop-win? It is natural to speculate whether $m_k = n(k, 5)$ for $k \geq 4$. It seems reasonable to expect that this is true at least for small values of k. It is known that $n(4, 5) = 19$, $n(5, 5) = 30$, $n(6, 5) = 40$ and $n(7, 5) = 50$. Do any of these cages attain the analogous m_k? More generally, we can ask the same question for large k: is m_k achieved by a $(k, 5)$-cage?

3. Graph genus

With the analogy of the chromatic number in mind, what can be said on bounds on the cop number in planar graphs? This was settled early on by Aigner and Fromme [1].

Theorem 5 ([1])**.** *If G is a planar graph, then $c(G) \leq 3$.*

The idea of the proof of Theorem 5 is to increase the *cop territory*; that is, a set vertices S such that if the robber moved to S, then he would be caught. Hence, the number of vertices the robber can move to without being caught is eventually is reduced to the empty set, and so the robber is captured. While their proof is indeed elegant, it is not easy to follow. We wrote a proof which hopefully is easier to digest in Chapter 4 of [17] (based on ideas of Brian Alspach and Boting Yang).

The *genus* of a graph is the smallest integer n such that the graph can be drawn without edge crossings on a sphere with n handles. Note that a planar graph has genus 0. Less is known about the cop number of graphs with positive genus, and this provides our second major conjecture on the topic. The main conjecture in this area is due to Schroeder, and this conjecture I think deserves to be better known. In [40], Schroeder conjectured that if G is a graph of genus g, then $c(G) \leq g + 3$. Quilliot [39] proved the following.

Theorem 6 ([39])**.** *If G is a graph of genus g, then $c(G) \leq 2g + 3$.*

In the same paper where his conjecture was stated, Schroeder showed the following.

Theorem 7 ([40]). *If G is a graph of genus g, then*

$$c(G) \leq \left\lfloor \frac{3g}{2} \right\rfloor + 3.$$

Theorem 7 implies the following.

Corollary 8 ([40]). *If G is a graph that can be embedded on a torus, then $c(G) \leq 4$.*

We do not know much about the planar graphs with cop numbers 1, 2, or 3. As cop-win graphs have a dismantling structure, that might help to classify the planar cop-win graphs but there is no success yet on that front.

4. Algorithms

We now describe a major conjecture on Cops and Robbers that was recently settled. Indeed, it is not everyday that one of your post-docs comes into your office claiming to have proven a 20 year old conjecture! We were lucky enough to have William Kinnersley as a post-doctoral fellow for two years starting in 2012. William came to Ryerson University having just completed his doctoral studies under Doug West's supervision, and he had a keen interest in the analysis of games played on graphs. While he had not worked much with Cops and Robbers before he came to Ryerson, the Cops and Robbers Principle was in effect, and he quickly delved into the topic.

EXPTIME is the class of decision problems solvable in exponential time. A decision problem is **EXPTIME**-complete if it is in **EXPTIME**, and for every problem in **EXPTIME** there is a polynomial-time algorithm that transforms instances of one to instances of the other with the same answer. William proved that computing the cop number is **EXPTIME**-complete. Before going further to discuss this, let us formalize things and consider the following two graph decision problems.

k-COP NUMBER: Given a graph G and a positive integer k, is $c(G) \leq k$?

k-FIXED COP NUMBER: Let k be a fixed positive integer. Given a graph G, is $c(G) \leq k$?

The main difference between the two problems is that in k-COP NUMBER the integer k may be a function of n, and so grows with n. In k-FIXED COP NUMBER, k is fixed and not part of the input, and so is independent of n.

The following result has been proved several times independently in the literature on the topic.

Theorem 9 ([6, 12, 29])**.** *The problem k-FIXED COP NUMBER is in **P**.*

If k is not fixed (and hence, can be a function of n), then the problem becomes less tractable.

Theorem 10 ([22])**.** *The problem k-COP NUMBER is **NP**-hard.*

Theorem 10 is proved in Fomin et al. [22] by using a reduction from the following well-known **NP**-complete problem:

DOMINATION: Given a graph G and an integer $k \geq 2$, is there a dominating set in G of cardinality at most k?

Goldstein and Reingold [26] proved that it is **EXPTIME**-complete to compute the k-COP NUMBER problem assuming the initial position of the cops and robber is given as part of the input. They also conjectured in [26] that k-COP NUMBER is **EXPTIME**-complete. Kinnersley settled this conjecture in a recent tour de force [30], using a series of non-trivial reductions.

Note that Theorem 10 does not say that k-COP NUMBER is in **NP**; that is an open problem! There is little research on the optimal running times for polynomial time algorithms to test if a graph has a small cop number such as 1, 2, or 3.

5. Variations

As you might expect, there are countless variations of the game of Cops and Robbers. Usually (though not always) such variations provide more complications than those found in the original game. One could play the game by giving the cops more power; in this direction, we studied the game of distance k Cops and Robbers [11, 12], where cops can capture the robber if it is within distance k. We could speed up the robber [24], allow the robber to capture a cop [13], or make the robber invisible [19, 20] (see Chapter 8 of [17] for more on these and other variants). We could also play on infinite graphs [15, 28], where many results from the finite landscape dramatically change.

We mention one variation in particular: the game of *Lazy Cops and Robbers*. This game is played in a similar fashion to Cops and Robbers, but only one cop may move at a time. Hence, Lazy Cops and Robbers is a game more akin to chess or checkers. The analogous parameter is the *lazy cop number*, written $c_L(G)$. Our knowledge of properties of the lazy cop number is limited, but in some cases its value is much larger than the classical cop number.

This game and parameter was first considered by Offner and Ojakian [35]. For hypercubes, it was proved in [32] that $c(Q_n) = \lceil \frac{n+1}{2} \rceil$. In contrast, the following holds for the lazy cop number.

$$2^{\lfloor \sqrt{n}/20 \rfloor} \leq c_L(Q_n) = O(2^n \log n / n^{3/2}). \tag{1}$$

A recent result of [5] improves the lower bound in (1).

Theorem 11 ([5])**.** *For all* $\varepsilon > 0$*, we have that*

$$c_L(Q_n) = \Omega\left(\frac{2^n}{n^{5/2+\varepsilon}}\right).$$

Thus, the upper and lower bounds on $c_L(Q_n)$ differ by only a polynomial factor. The proof uses the probabilistic method coupled with a potential function argument. It is an open problem to find the exact asymptotic order of $c_L(Q_n)$. The behaviour of the lazy cop number on planar graphs, or, more generally, graphs of higher genus is also not well understood.

Cops and Robbers represent the tip of the iceberg of what are called *vertex-pursuit games*, *graph searching*, or *good guys vs bad guys games* (the latter phrase was coined by Richard Nowakowski). A tough but fun problem in this general setting is on Firefighting in the infinite plane. Consider an infinite hexagonal grid. Every vertex is either on fire, clear, or protected. Initially, all vertices are clear. In the first round, fire breaks out on one vertex. In every round, a cop or *firefighter* protects one vertex which is not yet on fire. The fire spreads in the next round to all clear neighbors of the vertices already on fire. Once a vertex is on fire or is protected it permanently remains in that state. Note that unlike Cops and Robbers, the firefighter does not play on the graph, but can teleport anywhere it likes. Further, the fire the mindlessly spreads where it can.

Two firefighters can protect vertices so that the fire only burns two vertices in the hexagonal grid. It is not known if *one* firefighter can arrange things so the fire burns only *finitely* many vertices. In other words, can one firefighter build a wall containing the fire to a finite subgraph of the grid? It is conjectured that this is indeed impossible.

6. Photo and Bio

Anthony Bonato's research is in graph theory, focusing on vertex pursuit games and complex networks. He has authored over 90 publications with over 50 co-authors. His books *A Course on the Web Graph* (2008) and *The Game of Cops and Robbers on Graphs* (2011, joint with

R. Nowakowski) were published by the American Mathematical Society. He has delivered over 30 invited addresses at international conferences in North America, Europe, China, and India, and has supervised numerous graduate students and post-docs. Bonato is currently the Associate Dean, Students and Programs in the Yeates School of Graduate Studies, Editor-in-Chief of the journal *Internet Mathematics*, and editor of the journal *Contributions to Discrete Mathematics*.

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