$$T = \min \left\{ t : \exists \text{ some } j \in \{1, \dots, k\} \text{ st } X_t^j = Y_t \right\}$$

Assume $k = c(G)$.

$$ct(G) = \mathbb{E}(T \mid \text{both robber \& cops play optimally})$$

$$dct(G) = \mathbb{E}(T \mid k \text{ cops play optimally, robber drunk})$$

$$F = \frac{ct(G)}{dct(G)}$$ "the cost of drunkenness"

$F \geq 1$.

Lemma: Let G be a graph with $n = |V(G)|$ cop number $c(G)$ and let $\hat{T}$ be the time it takes $c(G)$ cops in the worst case to catch the robber.

Let Δ be the max degree of G, and D be the diameter of G.

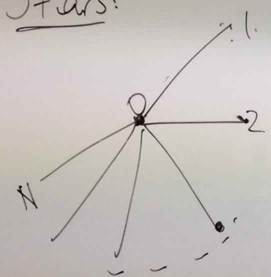
Then

$$ct(G) \leq (\hat{T} + D)(\Delta + 1)^{\hat{T}} n < \infty$$

Proof:

Cops start at opt position for classical case

Stars:



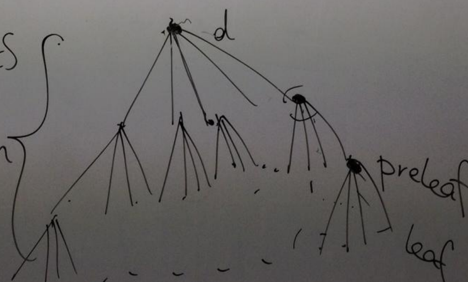
$$dct(G) = 1 \cdot \frac{N}{N+1} + 0 \cdot \frac{1}{N+1} = \frac{N}{N+1}$$

$$ct(G) = \frac{1}{N-1} \cdot 2 + \frac{1}{N-1} 4 + \ldots + \frac{1}{N-1}(2N-2) = N$$

$$f(G) = \underline{N+1 = n}$$

D-REGULAR TREES

$L = \log_d n$



$$ct(G) = \Theta(n \log n)$$

$$dct(G) = \Theta(n) \qquad F = \Theta(\log n)$$

$\frac{1}{N-1}(2N-2) = N$

<u>$ct(G)$:</u>

upper bound: strategy for cop

$\mathbb{P}[\text{cop is right in one round}]\ \Theta\left(\frac{1}{d^L}\right)$

<u>Lower bound:</u> strategy for robber

<u>$dct(G)$:</u> <u>upper bd:</u> cop waits at root

e_i = exp time when robber at level i

$$e_0 = 0$$

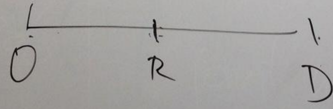
$$e_L = 1 + e_{L-1}$$

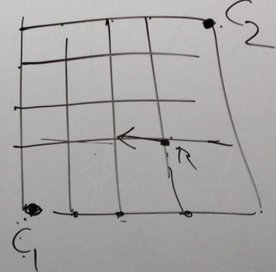
$$e_i = 1 + \frac{d}{d-1} e_{i+1} + \frac{1}{d-1} e_{i-1}$$

...ssword: RU guest 8

Grid $n = N \times N$ vertices

$dct(G) = \Theta(n)$

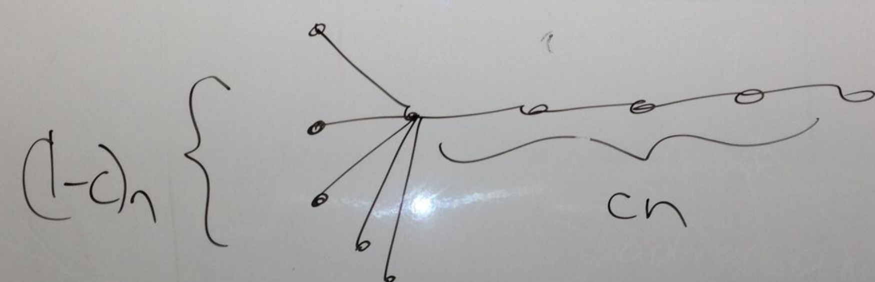




$ct(G) = \boxed{\Omega(n)}$

$= O(n^{1/2} 5^{\sqrt{n}} n)$ — $\boxed{\text{GAP}}$

Any rational value for $F \in [2, \infty)$ can be attained



Conj.: If $|V| \geq 2 \Rightarrow F \geq 2$

Q: What about irrational values of F?