Existence and multiplicity of positive radial solutions for singular superlinear elliptic systems in the exterior of a ball

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Abstract

We prove the existence and multiplicity of positive radial solutions to the nonlinear system

\[
\begin{aligned}
& -\Delta u_i = \lambda K_i(|x|)f_i(u_j) \quad \text{in } \Omega, \\
& d_i \frac{\partial u_i}{\partial n} + \tilde{c}_i(u_i)u_i = 0 \quad \text{on } |x| = r_0, \\
& u_i(x) \to 0 \quad \text{as } |x| \to \infty,
\end{aligned}
\]

for a certain range of \( \lambda > 0 \), where \( i, j \in \{1, 2\}, i \neq j \), \( \Omega = \{x \in \mathbb{R}^N : |x| > r_0 > 0\} \), \( N > 2 \), \( d_i \geq 0 \), \( K_i : [r_0, \infty) \to (0, \infty) \), \( \tilde{c}_i : [0, \infty) \to [0, \infty) \), \( f_i : (0, \infty) \to \mathbb{R} \) are continuous with possible singularity \( \pm \infty \) at 0 and satisfy a combined superlinear condition at \( \infty \).

*Joint work with D.D. Hai.*