Title: Portfolio optimization in a multidimensional structural default model with a focus on private equity.

Author: M. Escobar, Peter Hieber, Matthias Scherer.
Portfolio optimization in a multidimensional structural default model with a focus on private equity

Marcos Escobar
Department of Mathematics, Ryerson University, Toronto, 245 Church Street, Toronto, ON, Canada, email: escobar@ryerson.ca,

Peter Hieber
HVB-Institute for Mathematical Finance, Technische Universität München, Boltzmannstr. 3, 85748 Garching, Germany, email: peter.hieber@mytum.de,

Matthias Scherer
HVB-Institute for Mathematical Finance, Technische Universität München, Boltzmannstr. 3, 85748 Garching, Germany, email: scherer@tum.de.

Abstract
For risky investments, like private equity or hedge funds, default risk plays a prominent role. However, the accordant literature on portfolio optimization mostly disregards default risk and accordingly skewed return distribution. This paper presents a framework for a portfolio optimization including default risk. Default is modeled by means of a Merton- and Black-Cox-type structural model. On a portfolio level, the mean and covariance of the resulting return distribution can be derived analytically, allowing a classical mean-variance optimization. Since this optimization ignores tail risk, we additionally present a Monte-Carlo simulation for a mean-$CVaR$ optimization. The paper concludes with an application to unlisted private equity and compares its results to a model proposed by Hamada (1972) that does not consider default risk.

Keywords
Portfolio optimization; structural-default model; Black-Cox model; Merton model; private equity; default risk.