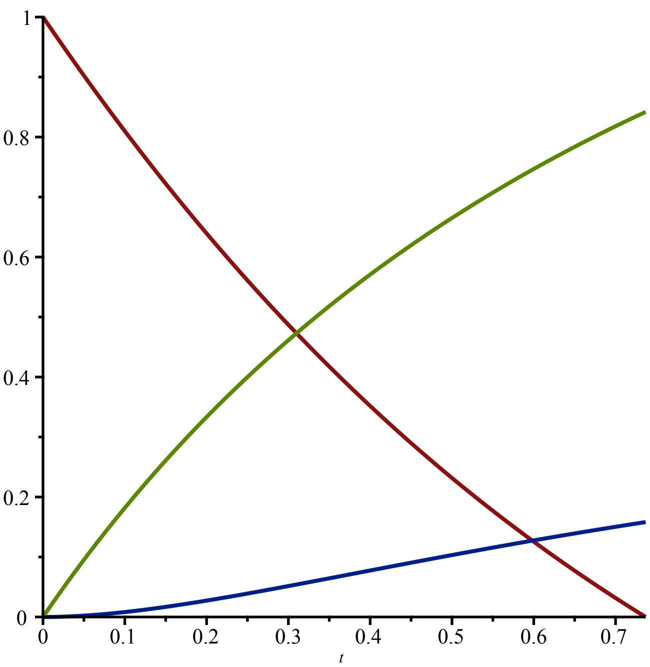
```
restart :
Digits := 32 :
k := 1100 :
# phase 1
systemde[1] := diff(x(t), t) = 2·(1 − x(t) + b(t)), diff(b(t), t) = x(t) − 4·b(t) :
ic[1] := x(0) = 0, b(0) = 0 :
dsol[1] := dsolve({systemde[1], ic[1]}, numeric, range = 0 ..1, output = listprocedure) :
xsol[1] := eval(x(t), dsol[1]) :
bsol[1] := eval(b(t), dsol[1]) :
final[1] := fsolve(1 − xsol[1](t) − bsol[1](t), t, 0 ..1, maxsols = 1, fulldigits)
```

$$final_1 := 0.73679318527482400390414074040212 \quad \textbf{(1)}$$

```
plot({xsol[1](t), bsol[1](t), 1- xsol[1](t) − bsol[1](t)}, t = 0 ..final[1])
```




```
for i from 2 to k do
 # phase i
 systemde[i] := diff(x(t), t) = 2·(1 − x(t) + r(t) + b(t)), diff(b(t), t) = (x(t) − 2·r(t) − 3·b(t))·i
     − b(t), diff(r(t), t) =- (x(t) − b(t))·(i − 1) − b(t)·(i − 1)/i − 2·r(t) :
 ic[i] := x(final[i − 1]) = xsol[i − 1](final[i − 1]), r(final[i − 1]) = bsol[i − 1](final[i − 1]),
     b(final[i − 1]) = 0 :
# print(ic[i]) :
dsol[i] := dsolve({systemde[i], ic[i]}, numeric, range = final[i − 1]..1.22, output = listprocedure) :
xsol[i] := eval(x(t), dsol[i]) :
bsol[i] := eval(b(t), dsol[i]) :
rsol[i] := eval(r(t), dsol[i]) :
final[i] := fsolve(rsol[i](t), t, final[i − 1]..1.22, maxsols = 1, fulldigits) :
if i < 11 or irem(i, 100) = 0 then print(i, final[i], xsol[i](final[i])) end if:
# print(plot({bsol[i](t), rsol[i](t)}, t = final[i − 1]..final[i]))
end do:
```

2, 0.89527964262941987865651589156679,

0.92604724544248228119165994251819576500537594066412764854829342663997183, 0.97168830125107456486642777316450,

0.95621720636704977164946022078493158056969331137127831916816956545179964, 1.01723251982893304696599011713379,

0.97079819680759707698098608253841769099134207119787045964429725955433775, 1.04762944129083064637343023195266,

0.97904198615979163525916648034778686037491483917071841231032643060306296, 1.06941773026114316474284134100217,

0.98418621020103528447842700870961742017344323296809227086022014248989597, 1.08582730599691771932570046675838,

0.98762363119324551663011867654719313547062137059562350816868313898862498, 1.09864430208744914053441195905099,

0.99003953008827098192104867883554962659409737569993637731815173556785039, 1.10893933020325071814810592878471,

0.991804872568675075476766703410976287459742689450761610790478042439661310, 1.11739427715071912238997444736381,

0.9931354294628557235182048618576540647082634329567726208399403356435294100, 1.19472034393325204543610862744831,

0.9999046283087728894874375627998275969340192575217596814797088851172293200, 1.19959721544685722458396368422071,

0.9999755987375174179727852981006833780515570689744415989912730347242320300, 1.20124050747524442943368600905871,

0.9999890685519212363024618019916741198103581296297433846398459855028960400, 1.20206557671689734249699353424121,

0.9999938263043862912626845588323928752304317862397056896535889572505982500, 1.20256172983584292501779892869781,

0.9999960392312982235905559550279769722225706033490790071885350927498112600, 1.20289296576475973858302998484311,

0.9999972449906962319058080604871215602212991606021274063067921262510759700, 1.20312979301728779587888452550561,

0.9999979735519322874046587569042838560124538545814076716373889386819299800, 1.20330753985808397006513137016271,

0.9999984471410615251186269367512024880708965646199119235347563623440050900, 1.20344586254685112511284702046461,

0.99999877221192046864687113516198409491072370856730138032734888845164411000, 1.20355656816033372106239685080381, 0.99999900494758343442918717097888772810890639889262014675175143313112051100, 1.2036471769254810651685426329510,

0.99999917727408671984622763422249365614213585540282916783834960789778 00 **(2)**

 # Regular precision: 1100,1.203648390,0.999999180961683

 # We get that after 1.20365n rounds there are less than 10^{-6}n unsaturated vertices

 # Original upper bound (with only red vertices)

restart :

$Digits := 32$:

$systemde := diff(x(t), t) = 2(1 - x(t) + z(t)), diff(z(t), t) = -\frac{2 \cdot z(t)}{1 - x(t)} \cdot (1 - x(t) + z(t)) - z(t) + x(t) - 2 \cdot z(t)$:

$ic := x(0) = 0, z(0) = 0$:

$dsol := dsolve(\{systemde, ic\}, numeric, range = 0..1.3, output = listprocedure)$:

```
Warning, cannot evaluate the solution further right of 1.2769439, 
maxfun limit exceeded (see ?dsolve,maxfun for details)
```

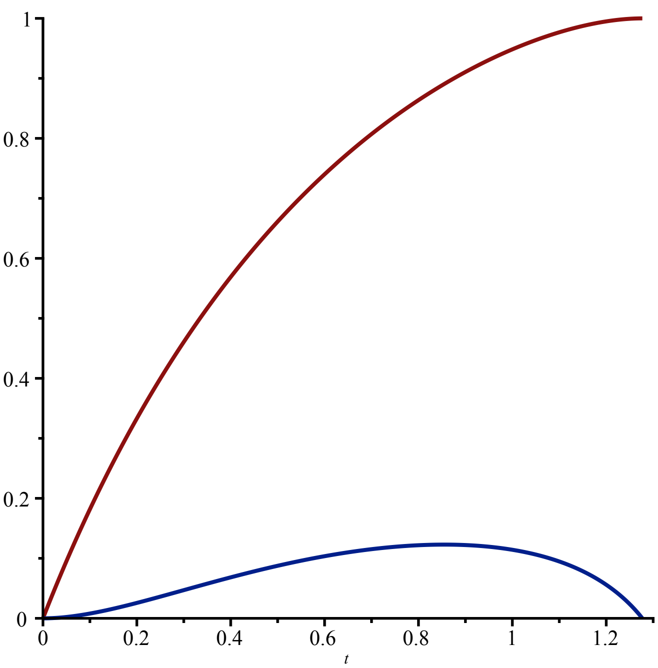
$xsol := eval(x(t), dsol)$:

$zsol := eval(z(t), dsol)$:

$fsolve\left(1 - xsol(t) - 10^{-14}, t, 0..1.3, maxsols = 1, fulldigits\right)$

1.2769436230120486100640087554592 **(3)**

$plot(\{xsol(t), zsol(t)\}, t = 0..1.3)$



 # Improved upper bound (*with red* **and** *blue vertices*). *The same DEs but different initial condition*

$restart:$

$Digits := 32:$

$$systemde := diff(x(t), t) = 2(1 - x(t) + z(t)), diff(z(t), t) = - \frac{2\cdot z(t)}{1 - x(t)}\cdot(1 - x(t) + z(t)) - z(t) + x(t) - 2\cdot z(t):$$

$ic := x(0) = 1 - 10^{-6}, z(0) = 0:$

$dsol := dsolve(\{systemde, ic\}, numeric, range = 0..0.025, output = listprocedure):$

Warning, cannot evaluate the solution further right of .15714004e-2, maxfun limit exceeded (see ?dsolve,maxfun for details)

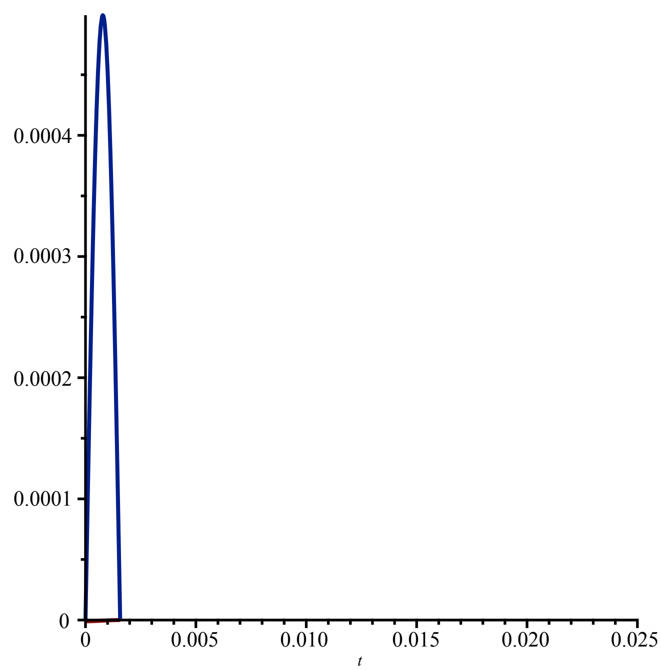
$xsol := eval(x(t), dsol):$

$zsol := eval(z(t), dsol):$

$fsolve\left(1 - xsol(t) - 10^{-14}, t, 0..0.025, maxsols = 1, fulldigits\right)$

$$0.0015703557663126582398910476302483 \quad (4)$$

$plot(\{xsol(t) - 1, zsol(t)\}, t = 0..0.025)$



we get the upper bound of

$1.20365 + 0.00158 + 0.00001$

$$1.20524 \quad (5)$$