RYERSON UNIVERSITY
DEPARTMENT OF MATHEMATICS

MTH714 - LOGIC & COMPUTABILITY - FINAL EXAM
December 11, 2007

INSTRUCTIONS

1. Duration: 3 hours
2. You are allowed two 8.5” × 11” formula sheets (two-sided).
3. Marks (out of 80) are shown in brackets.
4. Write your solutions in the space provided. If you need more space, use the back of the page. Indicate this fact on the original page, making sure that your solution cannot be confused with any rough work which may be there.
5. Do not separate the sheets.
6. Have your student card available on your desk.

Last Name (Print): ______________________________

First Name (Print): ______________________________

Student I.D. ______________________________

Signature ______________________________

Grade /80

[10 marks] **1.** What further truth values can be deduced from those given, if

(a) $v(p \wedge q) = T$ and $v(\neg(p \wedge q) \leftrightarrow (\neg p \rightarrow \neg q)) = F$?

(b) $v((p \rightarrow \neg q) \rightarrow (r \rightarrow q)) = F$?

[10 marks] **2.** Using semantic tableaux for propositional logic, check the validity of the formula:

$$(B \lor \neg A) \to \neg(A \to \neg B)$$

[10 marks] **3.** Show that the following predicate formula is not valid (p is a binary predicate symbol

$$[\forall x \forall y (p(x,y) \rightarrow p(y,x)) \wedge \forall x \forall y \forall z (p(x,y) \wedge p(y,z) \rightarrow p(x,z))] \rightarrow \forall x p(x,x)$$

by finding a counterexample.

[10 marks] **4.** Given is the proof of the valid formula

$$\vdash \forall x(B(x) \rightarrow C(x)) \rightarrow (\forall x B(x) \rightarrow \forall x C(x))$$

in the Hilbert proof system $\mathcal{H}$ for predicate logic.

Step	Formula	Justification
1.	$\forall x(A(x) \rightarrow B(x)), \forall x A(x) \vdash \forall x A(x)$	
2.	$\forall x(A(x) \rightarrow B(x)), \forall x A(x) \vdash A(a)$	
3.	$\forall x(A(x) \rightarrow B(x)), \forall x A(x) \vdash \forall x(A(x) \rightarrow B(x))$	
4.	$\forall x(A(x) \rightarrow B(x)), \forall x A(x) \vdash A(a) \rightarrow B(a)$	
5.	$\forall x(A(x) \rightarrow B(x)), \forall x A(x) \vdash B(a)$	
6.	$\forall x(A(x) \rightarrow B(x)), \forall x A(x) \vdash \forall x B(x)$	
7.	$\forall x(A(x) \rightarrow B(x)) \vdash \forall x A(x) \rightarrow \forall x B(x)$	
8.	$\vdash \forall x(A(x) \rightarrow B(x)) \rightarrow (\forall x A(x) \rightarrow \forall x B(x))$	

Provide appropriate justification for each step of the proof. You may use any fact about the system $\mathcal{H}$ mentioned in class or stated in the textbook.

[10 marks] **5.** Convert the following predicate formula into a PCNF:

$$\forall x(\forall y\, q(x, y) \rightarrow p(x)) \rightarrow \forall x p(x)$$

[10 marks] **6.** Using the algorithm(s) of your choice check whether the following pairs of literals are unifiable or not. For those pairs that are unifiable, find the most general unifier.

(a) $P(a, x, f(g(y)))$ and $P(y, f(z), f(z))$.

(b) $Q(a, x, f(g(x)))$ and $Q(z, h(z, u), f(u))$.

[10 marks] **7.** Using resolution in predicate logic, show that the following set of clauses is unsatisfiable:

$$\{\neg P(x) \vee \neg P(f(a)) \vee Q(y),\ P(y),\ \neg P(g(b,x)) \vee \neg Q(b)\}$$

[10 marks] **8.** (SLD-Resolution) Consider the following logic program

1. $p(y) \leftarrow q(x, y), r(y).$
2. $p(x) \leftarrow q(x, x).$
3. $q(x, x) \leftarrow s(x).$
4. $r(b).$
5. $s(a).$
6. $s(b).$

and assume that the computation rule always selects the leftmost literal for unification, while the search rule is top-to-bottom.

(a)Draw the SLD-tree for the given goal

$$\leftarrow p(x)$$

(b) What are the computed answer substitutions?