RYERSON UNIVERSITY

DEPARTMENT
OF
MATHEMATICS

MTH 607 **Midterm Test** **March 3, 2008**

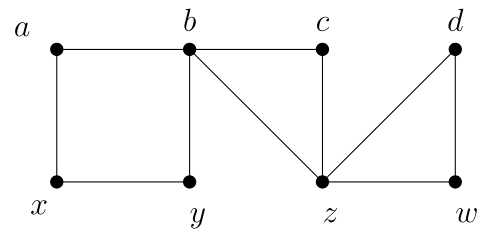
Total marks: 75 **Time allowed: 110 Minutes**

NAME (Print): ______________________ **STUDENT #:** ______________

Instructions:

- Verify that your paper contains 7 questions on 8 pages.
- You are allowed an $8\frac{1}{2} \times 11$ formula sheet written on both sides.
- No other aids allowed. Electronic devices such as calculators, cell-phones, pagers and Walkmans must be turned off and kept inaccessible during the test.
- Please keep your Ryerson photo ID card displayed on your desk during the test.
- In every question show all your work. The correct answer alone may be worth nothing.
- Delete all irrelevant and incorrect work because marks may be deducted for work which is misleading, irrelevant or incorrect, even if steps for a correct solution are also shown.
- Please write only in this booklet. Use of scrap paper or additional enclosures is not allowed. If you need more space continue on the back of the page, directing marker where the answer continues with a bold sign.
- All graphs in this test are assumed to be simple unless explicitly stated otherwise.

1.



G

For the graph G given above give the following, or explain why the graph does not have the given property. (Quote any theorems you use.)

2 Mk (a) Find the order, size, total degree and degree sequence.

Order	Size	Total Degree

Degree Sequence:

2 Mk (b) Find $\kappa(G)$, $\lambda(G)$, $\delta(G)$, $\Delta(G)$.

$\kappa(G)$	$\lambda(G)$	$\delta(G)$	$\Delta(G)$

2 Mk (c) Use the table below to give the adjacency **table** of G.

a	
b	
c	
d	
x	
y	
z	
w	

2 Mk (d) Draw a spanning tree of G.

2 Mk

(e) Where possible find an articulation point or a bridge. If it is not possible, briefly explain why not.

2 Mk

(f) Draw the induced subgraph of $\{y, b, c, d, w\}$.

2 Mk

(g) List the members of $[\{a, x, d\}, \{b, y, z\}]$.

2 Mk

(h) A bipartition of G.

2 Mk

(i) Label the vertices of G according to a Depth First Search, starting from the vertex x.

2. For each of the following either explain why the specified graph cannot exist (quote any theorems you use), or draw a graph with the given property.

3 Mk (a) A graph with degree sequence 3, 2, 2, 2, 1.

3 Mk (b) A graph with degree sequence 3, 2, 2, 1, 1.

3 Mk (c) A graph which is 3-connected, but not 3-edge connected.

3 Mk (d) A graph on 4 vertices that is isomorphic to its complement.

3 Mk (e) A 3-regular tree of height 2.

3 Mk (f) A bipartite graph which contains K_4.

3 Mk (g) A 2-connected subgraph containing a bridge.

3 Mk (h) For some $n \geq 4$, a simple $(n-2)$-regular disconnected graph of order n.

3. Draw the following:

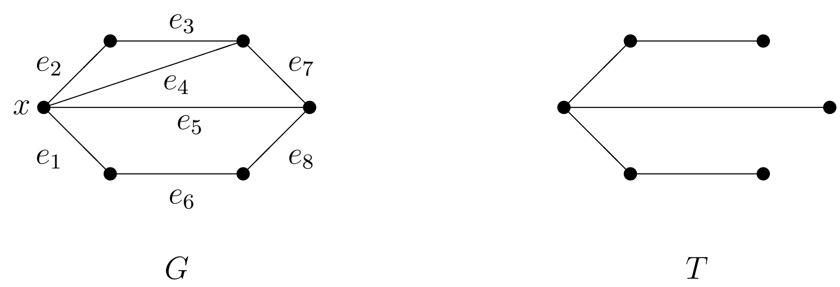
2 Mk (a) $\overline{P_5}$, the complement of P_5.

(b) $K_{3,2} \cup K_4$, the union of $K_{3,2}$ with K_4.
2 Mk (You may consider the vertex sets to be disjoint.)

2 Mk (c) $C_4 + K_2$, the join of C_4 with K_2.

2 Mk (d) $K_2 \times C_3$, the Cartesian product of K_2 with C_3.

4.



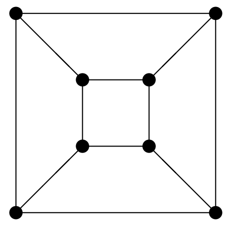
A graph G and a spanning tree T of G are given above.

2 Mk (a) Draw co$-T$.

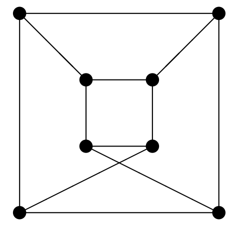
4 Mk (b) Give the Fundamental Cut sets of G with respect to T,

2 Mk (c) Find a cut set of G which is not fundamental, and express it as the ring sum of two fundamental cut sets.

5. Consider the graphs G and H given below.



G



H

4 Mk

(a) Find an isomorphic invariant to show that G and H are not isomorphic.

3 Mk

(b) Label the graph G above with bit strings of length 3 to show that $G \cong Q_3$. (Recall that Q_3 is the graph whose vertices are labeled by bit-strings of length 3 and there is an edge between to vertices if the strings that label them differ in exactly one place.)

4 Mk

6. Show that if a graph G has and odd degree vertex, then there are two vertices u and v both of odd degree which have a $uv-$path between them.

6 Mk

7. Show that a tree with a vertex of degree k contains at least k leaves. (Hint: use the Unique Tree Path Theorem.)