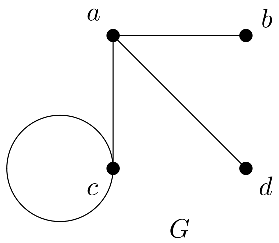
F06 MTH 607 Midterm Solutions

1.



G

For the graph G given above give the following, or explain why the graph does not have the given property. (Quote any theorems you use.)

(a) The degree sequence. $1, 1, 3, 3$

(b) The adjacency list.
$$\begin{array}{ll} a: & b, c, d \\ b: & a \\ c: & a, c \\ d: & a \end{array}$$

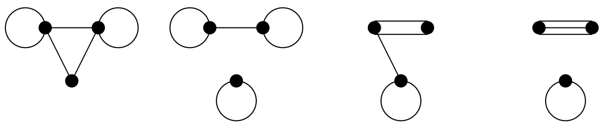
(c) A bipartition. *This graph cannot have a bipartition as it contains a loop.*

(d) An articulation point. a

2. For each of the following either explain why the specified graph cannot exist (quote any theorems you use), or draw a graph with the given property.

   (a) A graph with degree sequence 2, 3. *Such a graph cannot exist as the total degree is odd. Alternately, it has an odd number of vertices of odd degree.*

   (b) A graph with degree sequence 2, 3, 3. One of the following:

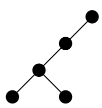


   (c) A simple bipartite graph on 6 vertices containing an Eulerian circuit and a Hamiltonian circuit. C_6

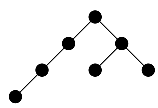
   (d) A 3−regular graph on 4 points. K_4

   (e) A tree with 7 vertices and 7 edges. *Every tree has on n points has $n - 1$ edges, so no such tree.*

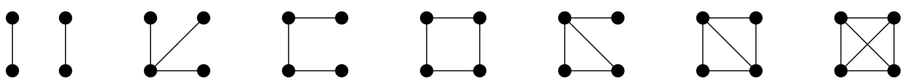
   (f) A binary tree of height 3 with 3 leaves.



OR

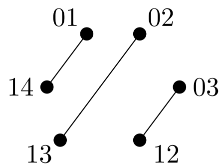


3. List all simple graphs with no isolated vertices on 4 points, up to isomorphism.



4. Let $P(n, t)$ be the graph whose vertices are labeled by sets of size t from an n set. Two vertices are adjacent if and only if the two sets that label them do not intersect. (The Peterson graph is $P(5, 2)$.)

   (a) Draw $P(4, 2)$.
   Take the 4-set $\{0, 1, 2, 3\}$, all the 2-sets from this 4-set are: $\{0, 1\}, \{0, 2\}, \{0, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$, which I will write as $01, 02, 03, 12, 13, 23$ respectively.



   (b) List the connected components of $P(4, 2)$.

$$\{01, 14\},\ \{02, 13\},\ \{03, 12\}.$$

   (c) Either provide a bipartition of $P(4, 2)$ or explain why one does not exist.

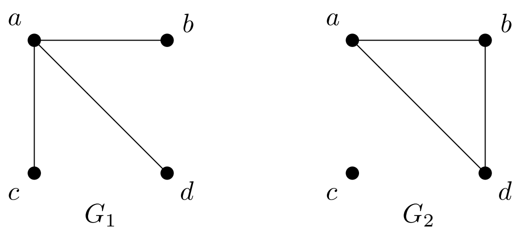
$$\{01, 02, 03\}, \{12, 13, 14\}\text{There are others.}$$

5. Prove that a connected graph G is a tree if and only if every edge of G is a cut-edge. (i.e. $G - e$ is disconnected for every $e \in E(G)$.)

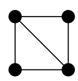
   ($\Rightarrow$) Let G be a tree with $e \in E(G)$, $e = x\,y$.
   Since G is a tree there is a unique $xy-$path (e), so $G - e$ is not connected.

   ($\Leftarrow$) Let G be a connected graph such that every edge is a cut edge.
   Consider two vertices $u, v \in V(G)$, since G is connected there must be at least one $uv-$path P.
   If there was another $uv-$path Q no edge of P would be a cut edge.
   Which contradicts the assumption that every edge is a cut edge.

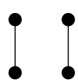
6. For this question let G_1 and G_2 be as given below.



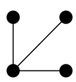
   (a) Draw $G_1 \cup G_2$.



   (b) Draw $G_1 \oplus G_2$.



   (c) Show that the complement of G_2 is isomorphic to G_1.

   The complement of G_2 is  . Take any bijective map from G_1 to the complement of G_2 which maps a f and $\{b, c, d\}$ to $\{a, b, d\}$.