RYERSON UNIVERSITY

DEPARTMENT
OF
MATHEMATICS

MTH 607 | **Midterm Test I** | **October 20, 2006**

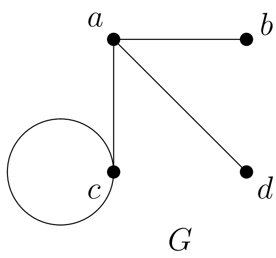
Total marks: 50 | **Time allowed: 110 Minutes**

NAME (Print): ______________________ **STUDENT #:** ______________

Instructions:

- Verify that your paper contains 6 questions on 6 pages.
- You are allowed an $8\frac{1}{2} \times 11$ formula sheet written on both sides.
- No other aids allowed. Electronic devices such as calculators, cell-phones, pagers and Walkmans must be turned off and kept inaccessible during the test.
- Please keep your Ryerson photo ID card displayed on your desk during the test.
- In every question show all your work. The correct answer alone may be worth nothing.
- Delete all irrelevant and incorrect work because marks may be deducted for work which is misleading, irrelevant or incorrect, even if steps for a correct solution are also shown.
- Please write only in this booklet. Use of scrap paper or additional enclosures is not allowed. If you need more space continue on the back of the page, directing marker where the answer continues with a bold sign.

1.



For the graph G given above give the following, or explain why the graph does not have the given property. (Quote any theorems you use.)

2 Mk (a) The degree sequence.

2 Mk (b) The adjacency list.

2 Mk (c) A bipartition.

2 Mk (d) An articulation point.

2. For each of the following either explain why the specified graph cannot exist (quote any theorems you use), or draw a graph with the given property.

3 Mk

(a) A graph with degree sequence 2, 3.

3 Mk

(b) A graph with degree sequence 2, 3, 3.

3 Mk

(c) A simple bipartite graph on 6 vertices containing an Eulerian circuit and a Hamiltonian circuit.

3 Mk

(d) A 3−regular graph on 4 points.

3 Mk

(e) A tree with 7 vertices and 7 edges.

3 Mk

(f) A binary tree of height 3 with 3 leaves.

5 Mk

3. List all simple graphs with no isolated vertices on 4 points, up to isomorphism.

4. Let $P(n,t)$ be the graph whose vertices are labeled by sets of size t from an n set. Two vertices are adjacent if and only if the two sets that label them do not intersect. (The Peterson graph is $P(5,2)$.)

   2 Mk (a) Draw $P(4,2)$.

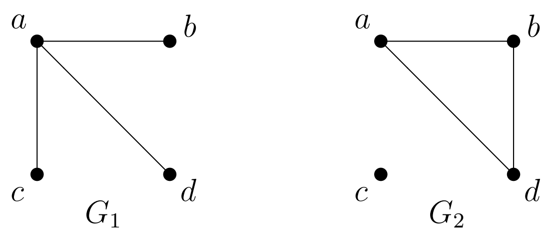
   2 Mk (b) List the connected components of $P(4,2)$.

   2 Mk (c) Either provide a bipartition of $P(4,2)$ or explain why one does not exist.

5. Prove that a connected graph G is a tree if and only if every edge of G is a cut-edge. (i.e. $G - e$ is disconnected for every $e \in E(G)$.)

   6 Mk

6. For this question let G_1 and G_2 be as given below.



2 Mk (a) Draw $G_1 \cup G_2$.

2 Mk (b) Draw $G_1 \oplus G_2$.

3 Mk (c) Show that the complement of G_2 is isomorphic to G_1.