RYERSON UNIVERSITY

DEPARTMENT
OF
MATHEMATICS

MTH 607 | **Final Exam** | **December 12, 2006**

Total marks: 80 | **Time allowed: 3hrs**

NAME (Print): ______________________________ **STUDENT #:** ____________________

Instructions:

- Verify that your paper contains 5 questions on 7 pages.
- You are allowed an $8\frac{1}{2} \times 11$ formula sheet written on both sides.
- No other aids allowed. Electronic devices such as calculators, cell-phones, pagers and Walkmans must be turned off and kept inaccessible during the test.
- Please keep your Ryerson photo ID card displayed on your desk during the test.
- In every question show all your work. The correct answer alone may be worth nothing.
- Delete all irrelevant and incorrect work because marks may be deducted for work which is misleading, irrelevant or incorrect, even if steps for a correct solution are also shown.
- Please write only in this booklet. Use of scrap paper or additional enclosures is not allowed. If you need more space continue on the back of the page, directing marker where the answer continues with a bold sign.

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1. For each of the following either explain why the specified graph cannot exist or draw a graph with the given property. (Be sure to quote any theorems you use, either by name or stating them explicitly.),

4 Mk

(a) A graph with 3 blocks and 1 cut vertex.

4 Mk

(b) A graph, G which has an independent set of size 3 and an edge cover of size 3.

(c) A graph, G, on 10 vertices which has a perfect matching and an edge cover of size 4.

4 Mk

4 Mk

(d) A graph G which is 3-connected, but not 3-edge connected.

4 Mk (e) A graph G which is 2-connected and 3-edge connected.

4 Mk (f) A graph G with more cut edges than cut vertices.

4 Mk (g) A 2-colourable graph containing a triangle (C_3).

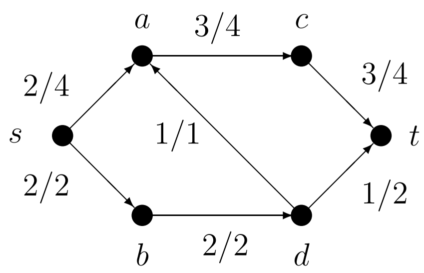
4 Mk (h) A 2-factorization of $K_{4,4}$.

2 Mk 2. (a) State Menger's Theorem for connectedness.

(b) Let G be a $5-$connected graph and let u, v and w be any three distinct vertices. Use Menger's Theorem to show that there are two cycles which have only the vertices u and v in common and don't use the vertex w.

5 Mk

3. A feasible flow f on a network N is given below.



2 Mk (a) Find val(f).

3 Mk (b) Find an $f-$augmenting path. Identify a forward and a backward edge in your path.

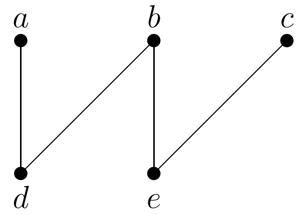
2 Mk (c) Identify a source sink cut, what is its capacity?

2 Mk (d) What is the maximum flow on this network? Show your reasoning.

3 Mk (e) Find the tolerance network N_f, label each edge of your graph with its associated tolerance.

2 Mk (f) Use your $f-$augmenting path from part (b) to produce a maximal flow. (Draw the resulting flow graph.)

4. Consider the bipartite graph G below:



9 Mk

Let $X = \{a, b, c\}$, $Y = \{d, e\}$ be the bipartition of G. Let M be the matching $\{be\}$ in G.

(a) Find an $M-$augmenting path in G with more than one edge in it.

(b) Use your $M-$augmenting path from part (a) to create an augmented matching M'. (List the edges of M'.)

(c) Either find an X saturating matching or use Hall's Theorem to explain why G does not have an X saturating matching (You **must** use Hall's Theorem, no other method is acceptable).

(d) Either find an Y saturating matching or use Hall's Theorem to explain why G does not have an Y saturating matching (You **must** use Hall's Theorem, no other method is acceptable).

7 Mk

5. Show that if a vertex u is a cut vertex of a graph G, then it is not a cut vertex of the complement of G, $\overline{G}$.