Trigonometric Review

Radians

In this course we use radians to measure angles.

The circumference of a circle radius r is $2\pi r$.
So in traversing a unit circle we travel a distance 2π.
If we traverse half way we travel a distance π.

Radians measure the distance around the unit circle we would travel.

Principal Angles

Quadrant I

Radians	Degrees
0	0
$\frac{\pi}{6}$	30
$\frac{\pi}{4}$	45
$\frac{\pi}{3}$	60

Quadrant II

Radians	Degrees
$\frac{\pi}{2}$	90
$\frac{2\pi}{3}$	120
$\frac{3\pi}{4}$	135
$\frac{5\pi}{6}$	150

Quadrant III

Radians	Degrees
π	180
$\frac{7\pi}{6}$	210
$\frac{5\pi}{4}$	225
$\frac{4\pi}{3}$	240

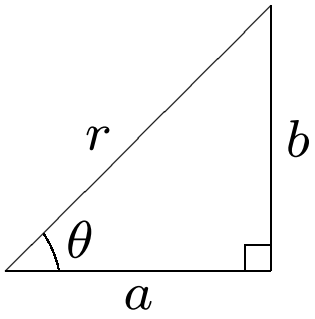
Quadrant IV

Radians	Degrees
$\frac{3\pi}{2}$	270
$\frac{5\pi}{3}$	300
$\frac{5\pi}{4}$	315
$\frac{11\pi}{6}$	330

$2\pi \sim 360^o$

Trigonometric Functions

Given a right angle triangle



$$\sin\theta = \frac{b}{r} = \text{Opposite over Hypotenuse}$$
$$\cos\theta = \frac{a}{r} = \text{Adjacent over Hypotenuse}$$
$$\tan\theta = \frac{b}{a} = \text{Opposite over Hypotenuse}$$

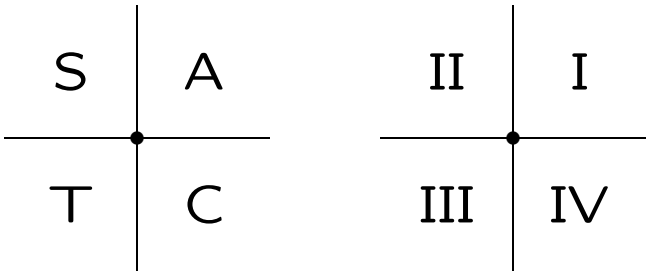
Note that $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

OHAHOA - Oh Heck Another Hour Of Algebra (sin, cos, tan).

Theorem 1 (Pythagoras' Theorem)
For any angle θ

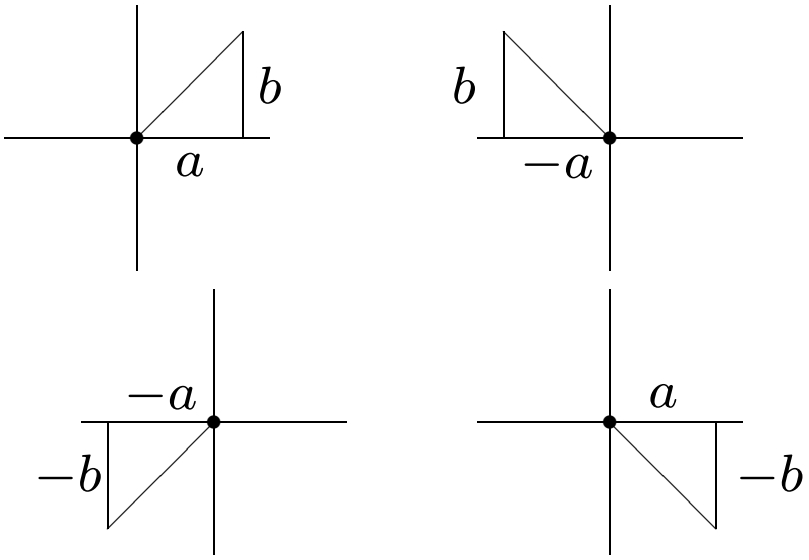
$$\cos^2\theta + \sin^2\theta = 1$$

The sign of sin, cos and tan in other quadrants is determined by the CAST rule:



C - cos is positive in Quadrant IV
A - All are positive in Quadrant I
S - sin is positive in Quadrant II
T - tan is positive in Quadrant III

Otherwise sin, cos and tan are negative.



Theorem 2 *for any angle* θ

$$\begin{aligned} \sin(-\theta) &= -\sin(\theta) \\ \tan(-\theta) &= -\tan(\theta) \\ \cos(-\theta) &= \cos(\theta) \end{aligned}$$

Note that adding 2π to an angle yeilds effectively the same angle (once more round the circle), so this does not affect the values of trigonometric functions. So for any value of θ:

$$\begin{aligned} \sin(2\pi + \theta) &= \sin(\theta) \\ \tan(2\pi + \theta) &= \tan(\theta) \\ \cos(2\pi + \theta) &= \cos(\theta) \end{aligned}$$

Principal Values

You are expected to know the following values for trig functions.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{2}$	1	0	$-$
π	0	-1	0
$\frac{3\pi}{2}$	1	0	$-$

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1

As well as the corresponding angles in the other three quadrants.

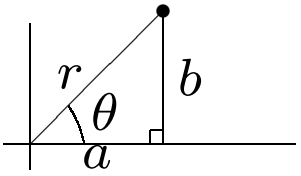
Calculating Angle with the Axis

Given the coordinate values (a, b) we wish to find the angle made with the $x-$axis, θ.

If $a \neq 0$

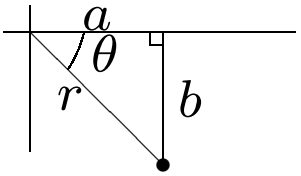
If $b \geq 0$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$



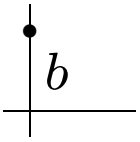
If $b < 0$

$$\theta = \pi + \tan^{-1}\left(\frac{b}{a}\right)$$



If $a = 0$

If $b > 0$, $\theta = \frac{\pi}{2}$



If $b < 0$, $\theta = \frac{3\pi}{2}$