Mx Mult^{c}

Oct 7 09

Mx Mult^{n} is **NOT** Commutative.

i.e. $AB \neq BA$.

Also can have 'Zero Divisors' i.e. Mt's A & B s.t $AB = 0$

eg 1) a) $\overset{A}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} \overset{B}{\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

b) $\underset{B}{\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}} \underset{A}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

$AB \neq BA$

2) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

3) I_2

I is the Identity of Mx Mult^{n}.

eg $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

Identity: $AI = IA = A$. for any Mx A.

Is it true that $(A+B)^2 = A^2 + 2AB + B^2$? **No!**

$$(A+B)^2 = (A+B)(A+B)$$
$$= A(A+B) + B(A+B) \quad \text{(Distributivity)}$$
$$= A^2 + AB + BA + B^2 \quad (\quad " \quad)$$

But we cannot guarentee $AB = BA$.

Distributivity $\quad A(B+C) = AB + AC \quad$ (order matters)

Matrix Transformat^{n}s

Given an $m \times n$ Mx A.

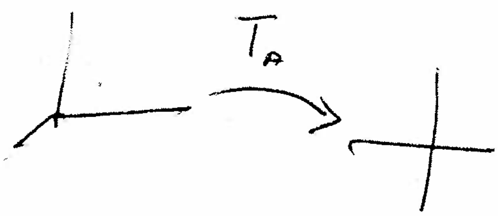
Then for every vector $\underline{x} \in \mathbb{R}^n$ we can define a vector $\underline{y} \in \mathbb{R}^m$ by $\underline{y} = A\underline{x}$

We call ~~this transformat^n~~ the transformat^n associated with A & write $T_A(\underline{x}) = A\underline{x}$

eg 1) $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ $\quad \underline{y} = A\underline{x} \quad y \in \mathbb{R}^2,\ x \in \mathbb{R}^3$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \end{pmatrix}$$



or
$$y_1 = x_1 + x_2$$
$$y_2 = x_2 + x_3$$

eg $T_A(1,1,1) = (2,2)$ $\qquad T_A(1,1,0) = (2,1)$

Notat^n We write $T_A : \mathbb{R}^n \to \mathbb{R}^m$

$\uparrow$ Maps $\qquad \uparrow$ to

2) $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\qquad T_A : \mathbb{R}^2 \to \mathbb{R}^3$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix}$$

$y_1 = x_1$
$y_2 = x_2$
$y_3 = 0$

Maps $\mathbb{R}^2$ onto the x, y plane $(z=0)$ of $\mathbb{R}^3$

If we fix $\underline{b} \in \mathbb{R}^m$ ask what values of $\underline{x} \in \mathbb{R}^n$ map to $\underline{b}$ via T_A

i.e. For what values of $\underline{x} \in \mathbb{R}$ is $A\underline{x} = \underline{b}$?

eg 1) Find $\underline{x} \in \mathbb{R}^3$ s.t. $A\underline{x} = \underline{b}$ where $\underline{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

& $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

i.e. $x_1 + x_2 = 1$
$x_2 + x_3 = 1$

This question actually asks for the sol^n to a system of eq^{n}s.

In general The eq^n $A\underline{x} = \underline{b}$ is actually a short form for an $m \times n$ system of eq^n's with coeff. mx A.

m eq^{n}s n unknowns

Mx Mult^n

Given 2 mx's A & B s.t. AB is defined.

Consider $\underline{y} = AB\underline{x}$ $\quad (\underline{y} = T_{AB}(\underline{x}))$

Let $\underline{z} = B\underline{x}$ $\quad (\underline{z} = T_B(\underline{x}))$

Then $\underline{y} = A\underline{z}$ $\quad (\underline{y} = T_A(\underline{z}) = T_A(T_B(\underline{x})))$

i.e. $T_{AB}(\underline{x}) = T_A(T_B(\underline{x}))$

So Matrix Mult$^{\underline{n}}$ corresponds to composit$^{\underline{n}}$ of maps

$$T_{AB}(\underline{x}) = T_A(T_B(\underline{x})) = (T_A \circ T_B)(\underline{x})$$

$$AB\underline{x} = A(B\underline{x})$$

<u>Linear Eq$^{\underline{n}}$'s Revisited</u>

S'pos we have an $m \times n$ mx A & a specific $\underline{b} \in \mathbb{R}^m$ $\underline{b} = (b_1, b_2, \ldots, b_m)$ Consider the eq$^{\underline{n}}$s $A\underline{x} = \underline{b}$ Where $\underline{x} = (x_1, \ldots, x_n)$ are unknowns.

<u>Notat$^{\underline{n}}$</u> A <u>Solut$^{\underline{n}}$</u> to $A\underline{x} = \underline{b}$ is the set of all $\underline{x} \in \mathbb{R}^n$ s.t. $A\underline{x} = \underline{b}$

The <u>Kernel</u> is the set of all $\underline{x} \in \mathbb{R}^n$ s.t. $A\underline{x} = \underline{0}$ i.e. The sol$^{\underline{n}}$ to the Homogeneous system $A\underline{x} = \underline{0}$

The Kernel is the set of points mapped to $\underline{0} \in \mathbb{R}^m$ by the transformat$^{\underline{n}}$ $T_A(\underline{x})$

Algebra of Mx's

Cancellat^{n} law work for matrix addit^{n} & scalar mult^{n}. But NOT Mx mult^{n}

1) $A+B=C \Rightarrow A=C-B$

2) $kA=B \Rightarrow A=\frac{1}{k}B \quad (k\neq 0)$

So we can do a certain amount of Algebra

eg Find the Mx X s.t.

$$2X+A=B$$

Where $A=\begin{pmatrix}-1&2&1\\0&1&-3\\2&0&0\end{pmatrix}$ $B=\begin{pmatrix}1&4&1\\6&-1&3\\-2&2&2\end{pmatrix}$

$$2X+\begin{pmatrix}-1&2&1\\0&1&-3\\2&0&0\end{pmatrix}=\begin{pmatrix}1&4&1\\6&-1&3\\-2&2&2\end{pmatrix}$$

$2X+A=B \Rightarrow 2X=B-A \Rightarrow X=\frac{1}{2}(B-A)$

$$B-A=\begin{pmatrix}1&4&1\\6&-1&3\\-2&2&2\end{pmatrix}-\begin{pmatrix}-1&2&1\\0&1&-3\\2&0&0\end{pmatrix}=\begin{pmatrix}2&2&0\\6&-2&6\\-4&2&2\end{pmatrix}$$

$$\frac{1}{2}(B-A)=\begin{pmatrix}1&1&0\\3&-1&3\\-2&1&1\end{pmatrix}$$

Problem Matix 'Division' is not well defined

We can define the 'inverse' of a square mx A, A^{-1} to be a mx s.t. $AA^{-1} = I = A^{-1}A$.

But Not all mx's have an inverse.

eg $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ has no inverse

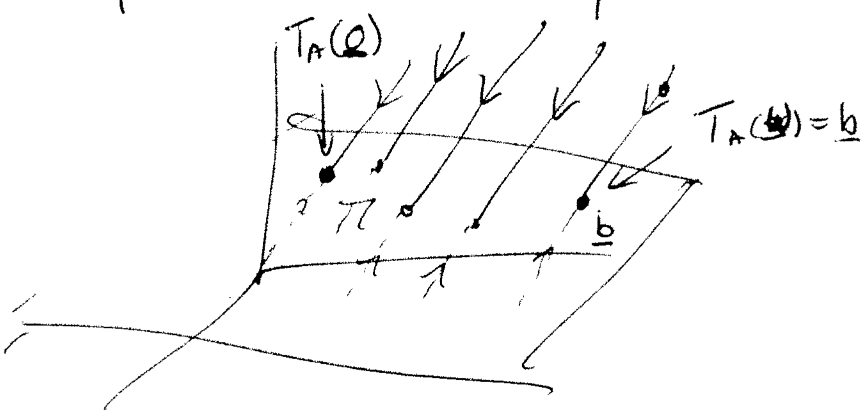
i.e. There is no Matrix B s.t. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ ~~so~~ $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ a+c & b+d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$a+c = 1$
$a+c = 0$ inconsistent.

If a mx has an inverse we call it invertable o/w it is non invertable.

S'pose a mx maps $\mathbb{R}^3$ to a plane



Recall $A\underline{h} = \underline{0}$ & $A\underline{w} = \underline{b}$
then $A(\underline{h} + \underline{w}) = \underline{b}$

$\underline{h} \in \ker(A)$ & $A\underline{w} = \underline{b}$
then $w + h$ is a sol^n of $A\underline{w} = \underline{b}$

If $\underline{h}$ is a sol^n to the homogeneous system $A\underline{x} = \underline{0}$ & $\underline{w}$ is a sol^n to $A\underline{x} = \underline{b}$ then $\underline{w} + \underline{h}$ is a sol^n to $A\underline{x} = \underline{b}$