$m \times n+1$ 30/09/09

eg. 1) $\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$ $r = 3$, $n = 3$, $m = 3$ $r \leq \min(m, n)$

Unique sol^n $r = n$

m = # of rows = # of eq^{n}s

n = # of cols $- 1$ = # of vars.

r = # of pivots in REF

$n - r$ = # of parameters in a sol^n.

2) a) $\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $r = 2$, $n = m = 3$ $z = t$ ....

b) $\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $y = t$ ....

3) $\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $r = 1$ 2-parameter sol^n,

4) $\begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ — No sol^n. $(0 = 1)$

Changing the constant part give sol^n

$$\begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & \boxed{0} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

5) $\left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & 1 & 2 & 1 \end{array}\right)$

6) a) $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $m = 4$, $n = 3$, $r = 3$ $(=n)$ unique sol^{n}

b) $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$ $m = 4$, $n = 3$, $r = 4$ No Sol^{n}.

Homogeneous Eq^{n}s

Th^{m} S'pose $\underline{h}$ is a sol^{n} to a Homogeneous system of Eq^{n}s. with coeff. Mx A. & $\underline{a}$ is a sol^{n} to the same system augmented with $\underline{b}$ then $\underline{h}+\underline{a}$ is also a sol^{n} to this latter system

eg $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ Homogeneous system:

$$x + 2y + z = 0$$
$$y = 0$$

1-parameter sol^{n}

Let $\underline{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ Augment with $\underline{b}$

$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightsquigarrow$

$$x + 2y + z = 1$$
$$y = 1$$

1-parameter sol^{n}.

Solve Homegeneous system.

Let $t \in \mathbb{R}$ $\quad z = t \quad y = 0 \quad x = -2t$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = t\begin{pmatrix}-1\\0\\1\end{pmatrix} = t\underline{v} \text{ (line through } O)$$

Now Consider System Augmented with $\underline{b}$

Th^m says $\quad \underline{a} = ~~(\ ,\ ,\ )~~$ is a sol^n.

$(-1, 1, 0)$

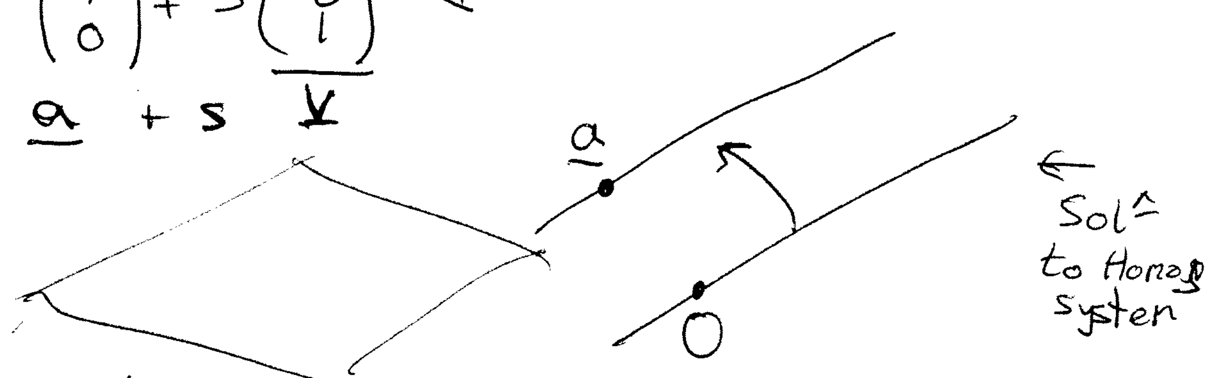
$\Rightarrow \underline{x} = \underline{a} + t\underline{v}$ is also a sol^n.

Now Solve $\quad \begin{matrix} x + 2y + z = 1 \\ y = 1 \end{matrix}$

Let $s \in \mathbb{R}$ $\quad s = z \quad y = 1, \quad x = 1 - 2 - s = -1 - s$

$$\underline{x} = \begin{pmatrix}-1\\1\\0\end{pmatrix} + s\begin{pmatrix}-1\\0\\1\end{pmatrix}$$

$$= \underline{a} + s\,\underline{v}$$



If we had used

$b = \begin{pmatrix}1\\1\\1\end{pmatrix}$ we have no sol^n.

Complex #'s

$0, 1, 2, \ldots$

Start with $\mathbb{N}$ - counting #'s (start at 0)

want to Solve $x+1=0$ $(x \in \mathbb{N})$

Invent a new # called -1

Defined to be s.t. $(-1)+1=0$

Have to extend our old # system $\mathbb{N}$ to $\mathbb{Z}$

$\mathbb{Z} = a(-1)$, or a $a \in \mathbb{N}$

$= \{ , \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}$ (The Integers)

Go on Want to solve $2x=1$ $[ax=b]$ $a,b \in \mathbb{Z}$

Sol^n $x=\frac{1}{2}$ q. Extend $\rightsquigarrow \mathbb{Q}$ rationals

$\mathbb{Q} \rightsquigarrow \mathbb{R}$.

Real #'s $\mathbb{R}$

Want to Solve $x^2+1=0$

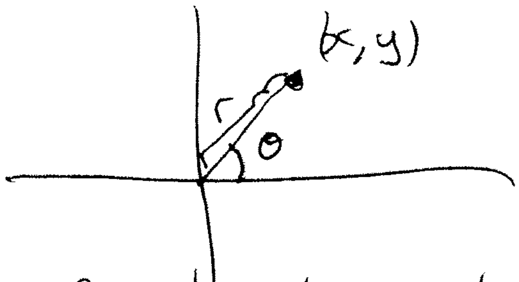
Invent a new # i s.t. $i^2=-1$

Extend our # system

$\mathbb{C} = \{ a+ib \mid a,b \in \mathbb{R} \}$ - Complex #'s.

Every complex # has a real part & an imaginary part

$\underline{z = x + iy}$ $\quad (x, y)$



Every Complex # can be represented by a point (vector) in the plane $\mathbb{R}^2$.

Could also represent z by $r \angle \theta$ (Polar Coords)

θ - angle with +ve x-axis

r - Length.

We write $z = re^{i\theta}$ $\quad (e = 2.78\ldots)$

eg 1) Take $z_1 = 1 + 2i$, $z_2 = 3 + i$

a) $z_1 + z_2 = 1 + 2i + 3 + i = 4 + 3i$

b) $z_1 z_2 = (1+2i)(3+i) = 3 + 2i^2 + 6i + i$

$$= \underset{(i^2=-1)}{3 - 2} + 7i = \underline{1 + 7i}$$

2) Find z where $z + (6+3i) = 1 + 2i$

$$z = 1 - 6 + (2-3)i = -5 - i$$

$$z = (1+2i) - (6+3i) = -5 - i$$

3) Find z where $\bar{z} + 1 - 2i = 3 - 3i$

$$\Rightarrow \bar{z} = 3 - 3i - (1 - 2i) = 2 - i$$

So $z = 2 + i$ $\quad (z = \bar{\bar{z}})$

4) Find real #'s x & y s.t. $xi + y(2-i) = 4+i$

LHS $= xi + 2y - yi = (x-y)i + 2y$

$\Rightarrow 2y = 4$ (Real part)

$x - y = 1$ (Im part)

$y = 2$ & $x = 3$

<u>Division</u>

z/u Given $z \in \mathbb{C}$ find $\frac{1}{z} \in \mathbb{C}$ s.t.

$$z \cdot \left(\frac{1}{z}\right) = 1$$

Consider $z \cdot \bar{z} = (x+yi)(x-iy) = x^2 + y^2 = \overset{\in \mathbb{R}}{|z|^2}$

So $z \cdot \frac{\bar{z}}{|z|^2} = 1$

Take $\frac{1}{z} = \frac{\bar{z}}{|z|^2} = \frac{1}{x^2+y^2}(x - iy)$

eg 1) Find $\frac{1}{3+4i} = \frac{1}{9+16}(3-4i) = \frac{3}{25} - \frac{4}{25}i$

2) Find $\frac{3+4i}{2+i} = (3+4i)\left(\frac{1}{2+i}\right) = (3+4i)\left(\frac{1}{2^2+1^2}(2-i)\right)$

$= (3+4i)\frac{1}{5}(2-i) = \frac{1}{5}(10+5i) = 2+i$

Note $(2+i)^2 = 3+4i$ | So $\frac{3+4i}{2+i} = 2+i$