Gauss

28/09/09

eg ~~xx~~ From Intro Find the intersect^n of the plane

$$x+y+z=1$$
$$x+2y-z=0$$

[2 Eq^{n}s in 3 vars]

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 0 \end{pmatrix} \qquad R_2 \to R_2 - R_1$$

$$\begin{array}{ccc} x & y & z \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & -1 \end{array}\right) \qquad [REF]$$

[$3-2=1$ parameter sol^n]

Let $t \in \mathbb{R}$ set $z=t$

$$y-2z=-1 \Rightarrow y=-1+2z=\underline{-1+2t}$$

$$x+y+z=1 \Rightarrow x=1-y-z=1-(-1+2t)-t$$

$$=1+1-2t-t$$

$$\underline{x=2-3t}$$

So $(x,y,z)=(2-3t,-1+2t,t)$

Or $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2-3t \\ -1+2t \\ t \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$$

line through $(2,-1,0)$ // to $(-3,2,1)$

2) Find the intersect^{n} of

$$\begin{aligned} x+2y-3z&=1\\ y+z&=1\\ x+y+z&=0 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 1\\ 0 & 1 & 1 & 1\\ 1 & 1 & 1 & 0 \end{array}\right) \qquad R_3 \to R_3 - R_1$$

$$\left(\begin{array}{cccc} 1 & 2 & -3 & 1\\ 0 & 1 & 1 & 1\\ 0 & -1 & 4 & -1 \end{array}\right) \qquad R_3 \to R_3 + R_2$$

$$\left(\begin{array}{cccc} 1 & 2 & -3 & 1\\ 0 & 1 & 1 & 1\\ 0 & 0 & 5 & 0 \end{array}\right)$$

$5z = 0 \Rightarrow z = 0$

$y+z=1 \Rightarrow (z=0)\ y=1$

$x+2y-3z=1 \Rightarrow x = 1-2y = 1-2 = -1$

$(x,y,z) = (-1,1,0)$

3)

$$x + 2y - 3z = 1$$
$$y + z = 1$$
$$x + 3y - 2z = 2$$

$$\begin{pmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 3 & -2 & 2 \end{pmatrix} \quad R_3 \to R_3 - R_1$$

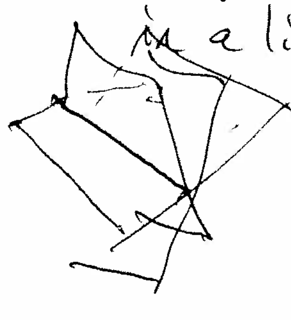
$$\begin{pmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \quad R_3 \to R_3 - R_2$$

$$\begin{pmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Let $t \in \mathbb{R}$ set $z = t$ $\quad y = 1 - t$

$$x = 1 - 2y + 3z = 1 - 2(1-t) + 3t$$
$$= 1 - 2 + 2t + 3t$$
$$= -1 + 5t$$

So $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$ – Three planes meeting in a line



4)

$$x + 2y - 3z = 1$$
$$y + z = 1$$
$$x + 3y - 2z = 0$$

$$\begin{pmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 3 & -2 & 0 \end{pmatrix} \quad R_3 \to R_3 - R_1$$

$$\begin{pmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix} \quad R_3 \to R_3 - R_2$$

$$\begin{pmatrix} 1 & 2 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

3^{rd} row $\quad 0 = -2$

No $\underline{\text{Sol}^n}$.

Notes:

1) No sol^n when ~~sol~~ a row reduces to $(0 \dots 0 | x)$. where x is non zero.

2) ~~#dk~~ If there is a sol^n # of parameters $= n - r$
where n = # of vars. = # cols of coeff. mx ← matrix
r = # of pivots in the REF. := $\underline{\text{rank}}$ of coeff. mx

5) For what values of ~~the~~ a is the folling system consistent?

$$x + 2y + 3z = -1$$
$$2x + y = 1$$
$$3x + 3y + 3z = a$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 2 & 1 & 0 & 1 \\ 3 & 3 & 3 & a \end{array}\right) \qquad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & -3 & -6 & 3 \\ 0 & -3 & -6 & a+3 \end{array}\right) \qquad R_2 \to -\tfrac{1}{3} R_2$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & -3 & -6 & a+3 \end{array}\right) \qquad R_3 \to R_3 + 3R_2$$

$$\left(\begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & a \end{array}\right)$$

Last row $0x + 0y + 0z = a \Rightarrow 0 = a$

System is consistent iff. $a = 0$

inconsistent o/w. ← otherwise

6) For what values of k does the following have
i) Unique Sol^n ii) ∞'ly many sol^n's iii) No Sol^n.

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (k^2-14)z = k+2.$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & k^2-14 & k+2 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 4R_1 \end{array}$$

$k^2-14+12$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & k^2-2 & k-14 \end{array}\right) \quad R_2 \to -\tfrac{1}{7}R_2$$

$$\left(\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/7 \\ 0 & -7 & k^2-2 & k-14 \end{array}\right) \quad R_3 \to R_3 + 7R_2$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/7 \\ 0 & 0 & k^2-16 & k-4 \end{array}\right)$$

Row 3 $\Rightarrow$ $(k^2-16)z = k-4$

$\left[z = \frac{k-4}{k^2-16} \Leftarrow k^2 \neq 16,\ k \neq \pm 4 \right]$

$\underline{k \neq \pm 4}$ $\Rightarrow$ $z = \frac{k-4}{k^2-16}$ — Unique Sol^n.

$\underline{k = 4}$ 3rd row becomes 0 0 0 0

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/7 \\ 0 & 0 & 0 & 0 \end{array}\right) \text{ — 1-parameter sol}^n.$$

(Case ii)

$\underline{k = -4}$ 3rd row becomes $(0\ 0\ 0 \,|\, -8)$ $0 = -8$

No Sol^n.

$$\boxed{\text{i) } k \neq \pm 4 \qquad \text{ii) } k = 4 \qquad \text{iii) } k = -4}$$