Upper & Lower Triangular Mxx. Oct 28 09

eg 1) $$\begin{pmatrix}1&2&3\\0&2&1\\0&0&1\end{pmatrix}\begin{pmatrix}2&1&1\\0&1&2\\0&0&1\end{pmatrix}=\begin{pmatrix}2&3&8\\0&2&5\\0&0&1\end{pmatrix}$$

2) $$\begin{pmatrix}1&0&0\\1&0&0\\3&1&1\end{pmatrix}\begin{pmatrix}2&0&0\\0&1&0\\1&1&2\end{pmatrix}=\begin{pmatrix}2&0&0\\2&0&0\\7&2&2\end{pmatrix}$$

3) $$\operatorname{diag}(1,2,3,4)\operatorname{diag}(2,1,2,3)=\operatorname{diag}(2,2,6,12)$$

$$\begin{pmatrix}1&0&0&0\\0&2&0&0\\0&0&3&0\\0&0&0&4\end{pmatrix}\begin{pmatrix}2&0&0&0\\0&1&0&0\\0&0&2&0\\0&0&0&3\end{pmatrix}=\begin{pmatrix}2&0&0&0\\0&2&0&0\\0&0&6&0\\0&0&0&12\end{pmatrix}$$

4) $D=\operatorname{diag}(1,2,3,4)$

$$D^{-1}=\operatorname{dig}\left(1,\tfrac{1}{2},\tfrac{1}{3},\tfrac{1}{4}\right)=\begin{pmatrix}1&0&0&0\\0&\frac{1}{2}&0&0\\0&0&\frac{1}{3}&0\\0&0&0&\frac{1}{4}\end{pmatrix}$$

Check

$$DD^{-1}=\begin{pmatrix}1&0&0&0\\0&2&0&0\\0&0&3&0\\0&0&0&4\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&\frac{1}{2}&0&0\\0&0&\frac{1}{3}&0\\0&0&0&\frac{1}{4}\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

5) $D=\operatorname{diag}(1,2,2,1)$

$$D^4=\operatorname{diag}(1^4,2^4,2^4,1^4)=\operatorname{diag}(1,16,16,1)$$

check

$$\left(\begin{pmatrix}1&0&0&0\\0&2&0&0\\0&0&2&0\\0&0&0&1\end{pmatrix}^2\right)=\begin{pmatrix}1&0&0&0\\0&4&0&0\\0&0&4&0\\0&0&0&1\end{pmatrix}^2=\begin{pmatrix}1&0&0&0\\0&16&0&0\\0&0&16&0\\0&0&0&1\end{pmatrix}$$

Fixed Points

Given a mx A we can define a map T_A by

$$\underline{y} = A\underline{x}$$

A fixed pt. is a pt which is fixed by this map.

i.e. $A\underline{x} = \underline{x}$

Note $\underline{0}$ is always a fixed pt of a mx trans.

Consider $A\underline{x} = \underline{x}$

$$\Rightarrow A\underline{x} - \underline{x} = \underline{0}$$

$$\Rightarrow A\underline{x} - I\underline{x} = \underline{0}$$

$$\Rightarrow (A-I)\underline{x} = \underline{0}$$

So Fixed pts of ~~the~~ A (T_A) are sol$^{\underline{n}}$s to the homogen eqs system $(A-I)\underline{x} = \underline{0}$

Corollary Fixed pts. are a subspace,

eg 1) Let $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ Find all the fixed pts of A.

Solve $(A-I)\underline{x} = \underline{0}$

$$A - I = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right) \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right) R_2 \leftrightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) R_2 \to -R_2 \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

Only sol$^{\underline{n}}$ is $\underline{x} = \underline{0}$

2) $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ Find the fixed pts of A

3

$A - I = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix} \xrightarrow{GE} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$x + y + z = 0$
$\cancel{x}+y = 0$
$y + z$

Let $t \in \mathbb{R}$ $z = t$ $y = -t$ $x = \cancel{?}\ 0$

Fixed Pts. are all of the form

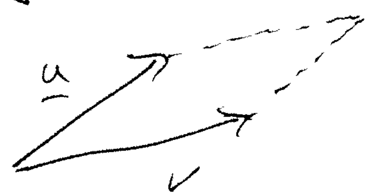
$\underline{x} = t \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ - line through O

Determinants

Geometric interpretat^{n} of Det. (in $\mathbb{R}^2$)

Given two non-parallel vectors $\underline{u}$ & $\underline{v}$

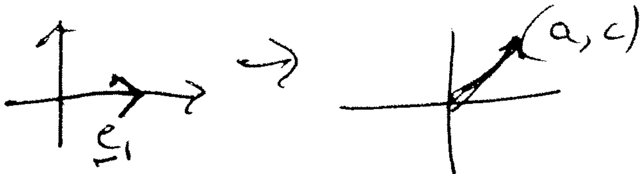
They 'map out' a parallelogram.



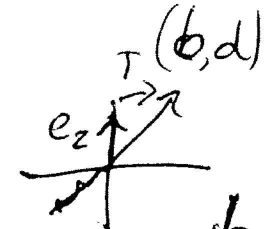
Th^{m} The area of the //gram 'mapped out' by $\underline{u} = (u_1, u_2)$ & $\underline{v} = (v_1, v_2)$ is $|u_1 v_2 - v_1 u_2|$

Consider the map defined by the mx $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$T_A(\underline{e}_1) = A\underset{\sim}{e}_1 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$$

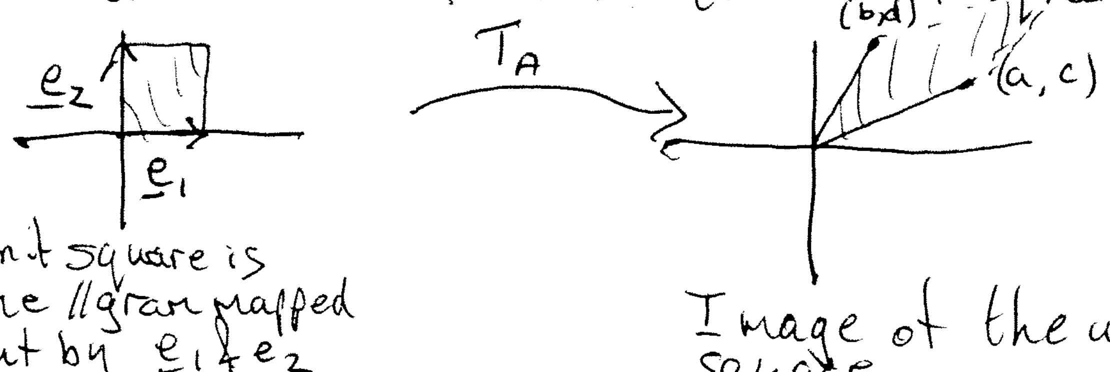


$$T_A(\underline{e}_2) = A\underline{e}_2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix}$$



Let $\underline{u} = T_A(\underline{e}_1) = \begin{pmatrix} a \\ c \end{pmatrix}$ & $\underline{v} = T_A(\underline{e}_2) = \begin{pmatrix} b \\ d \end{pmatrix}$

Consider the effect on the unit square



Unit square is the //gram mapped out by $\underline{e}_1$ & e_2

Image of the unit square

Area $= |ad - bc| = \det(A)$

~~eg~~

eg 1) Find $|A|$ where $A = \begin{array}{c} \begin{matrix} + & - & + & - \end{matrix} \\ \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix} \end{array}$

5

$$0\cancelto{0}{\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix}} - 1\begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix}$$

$$-1\left(1\cancelto{0}{\begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix}} - 1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1\cancelto{0}{\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix}}\right)$$

$$+2\left(1\cancelto{0}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}} - 0 + 1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}\right)$$

$$-3\left(1\cancelto{0}{\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}} - 0 + \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}\right)$$

$$= \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 3\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= 0$$

Same Q

Expand Down 3rd col.

[Always say what you're doing]

$$2\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 3 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 2\left(\cancelto{0}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}} + \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}\right) - \left(-\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}\right) = 0$$