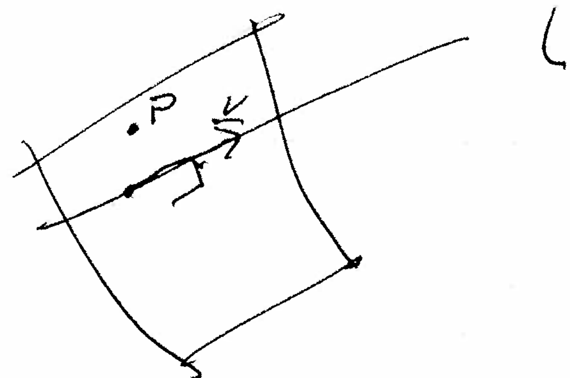
Finding The Eq^{n} of Planes in $\mathbb{R}^3$

23/09/09

eg 1) Find the Eq^{n} of the plane $\perp$ to the line L: $\underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{\vec{OQ}} + t\underbrace{\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}}_{\underline{v}}$ & through the Point $P = (1, 2, 1)$



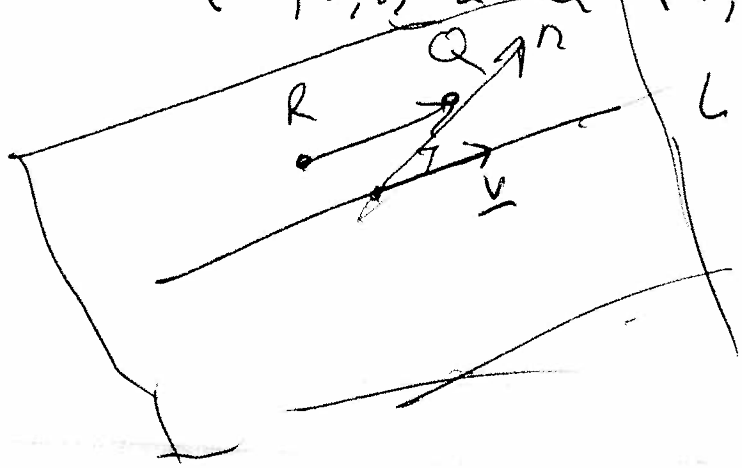
$\underline{v}$ is perpendicular or normal to the plane.

~~So~~ eg $\underline{v} = (1, 2, 0)$ so eq^{n} of the plane is $x + 2y + 0z = d$ $\quad x + 2y = d$

$d = \underline{n} \cdot \vec{OP} = (1, 2, 0) \cdot (1, 2, 1) = 1 + 4 = 5$

Eq^{n} is $\boxed{x + 2y = 5}$

2) Find the Plane // to L (above) & Through $R = (0, 1, 1)$ & $Q = (1, 1, 1)$



$\underline{v} = (1, 2, 0)$

$\vec{RQ} = (1, 0, 0)$

Want $\underline{n} = (n_1, n_2, n_3)$ s.t.

23/09/09

$\underline{n} \cdot \underline{v} = 0$ & $\underline{n} \cdot \overrightarrow{RQ} = 0$

$n_1 + 2n_2 = 0$ $\quad n_1 = 0 \quad \Rightarrow n_2 = 0$

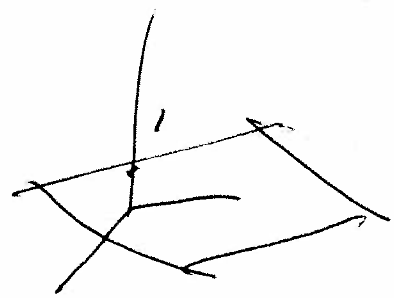
Any vector of the form $(0, 0, t)$ $t \in \mathbb{R}$ will do

Chose $\underline{n} = (0, 0, 1)$

$\therefore$ Plane has Eq^n $z = d$

$d = \underline{n} \cdot \overrightarrow{OR} = 1$

Eq^n of Plane is $\boxed{z = 1}$



3) Find the eq^n of the plane through the lines

$L_1: \underline{x} = \underset{P}{\begin{pmatrix}1\\1\\1\end{pmatrix}} + t \underset{\underline{v}}{\begin{pmatrix}1\\2\\0\end{pmatrix}}$ & $L_2: \underline{x} = \underset{Q}{\begin{pmatrix}2\\1\\2\end{pmatrix}} + s \underset{\underline{w}}{\begin{pmatrix}4\\-2\\5\end{pmatrix}}$

[P & Q lie in the plane & $\underline{v}$ & $\underline{w}$ are $\parallel$ to it]

[Want to find $\underline{n}$ $\perp$ to both $\underline{v}$ & $\underline{w}$.]

Let $\underline{n} = (n_1, n_2, n_3)$

s.t. $\underline{n} \cdot \underline{v} = 0$ $\quad \underline{n} \cdot \underline{w} = 0$

$n_1 + 2n_2 = 0$ $\quad 4n_1 \cancel{4} - 2n_2 + 5n_3 = 0$

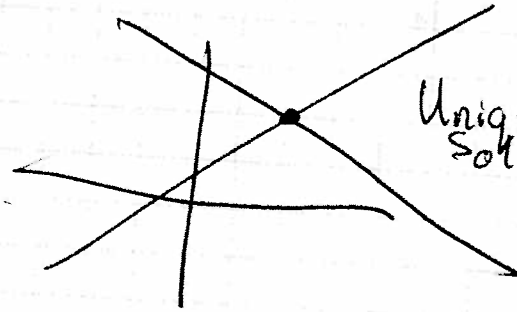
Sol^n $\underline{n} = (10, -5, -10)$

$d = \overrightarrow{OP} \cdot \underline{n} = -5$

$\boxed{10x - 5y - 10z = -5}$

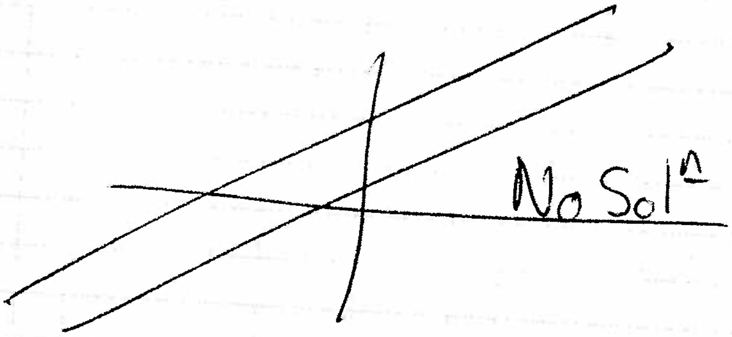
OR $\boxed{2x - y - 2z = -1}$

$\mathbb{R}^2$



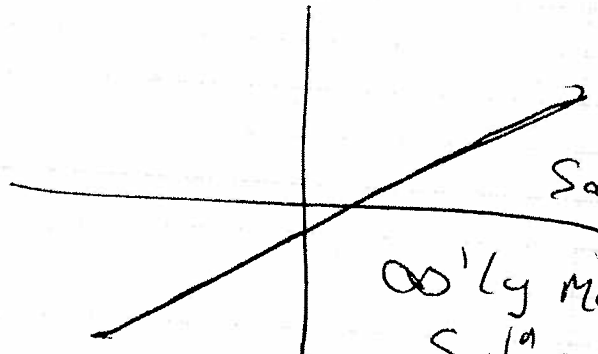
Unique Sol$^{\underline{n}}$ 2 lines meet in a pt

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$



No Sol$^{\underline{n}}$

Never meet

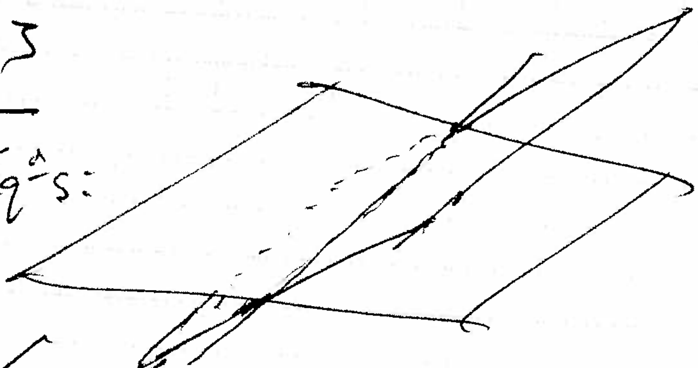


Same line

∞'ly many Sol$^{\underline{n}}$'s.

$\mathbb{R}^3$

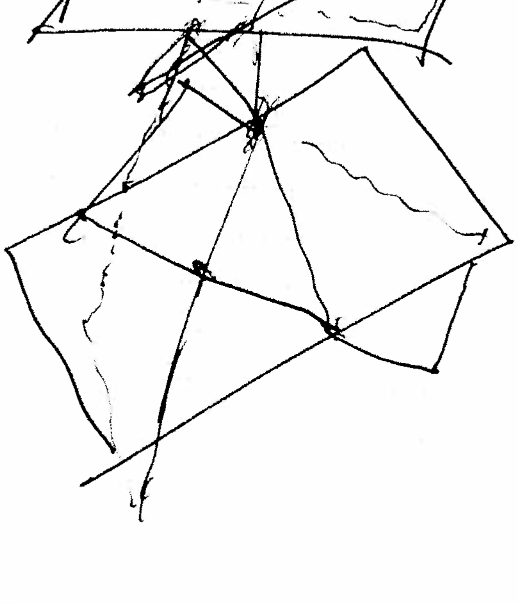
2 Eq$^{\underline{n}}$s:

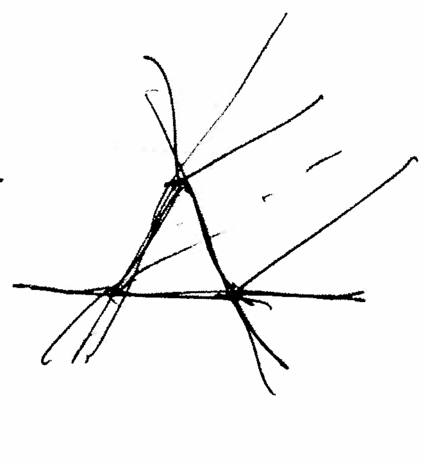


2 Planes meet in a line

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$

– 3 Planes meet in a pt.





– No Sol$^{\underline{n}}$

Systems of Equat^{n}s

eg Solve

$$x+y=3 \quad (1)$$
$$2x+3y=7 \quad (2)$$

$$\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & 3 & 7 \end{array}\right)$$

$(2) - 2\times(1)$

$$\begin{array}{r} 2x+3y=7 \\ -\ 2x+2y=6 \\ \hline 0x+\underline{y}=1 \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 1 \end{array}\right) \quad R_2 \to R_2 - 2R_1$$

$y=1 \Rightarrow x+1=3 \Rightarrow x=2.$

Add a multiple of one Row to another.
Mutiply a row by a constant.
Interchange 2 rows

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2)

$$x+y+z=0$$
$$2x+y+z=1$$
$$3x+2y+2z=1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 3 & 2 & 2 & 1 \end{array}\right)$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & -1 & -1 & +1 \end{pmatrix} \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & -1 & -1 & 1 \end{pmatrix} \quad R_2 \to -1R_2$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad R_3 \to R_3 + R_2$$

$$x+y+z=0$$
$$y+z=-1$$
$$0=0$$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left(\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 0 \\ 0 & \boxed{1} & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \end{array}$$

For each col. not containing a pivot set the corresponding variable to be a parameter

Let $t \in \mathbb{R}$ set $z = t$

So $y = -1+t$

$x = -y-z = -(-1+t) - t = 1-2t$

$$\begin{aligned} x &= 1-2t \\ y &= -1+t \\ z &= t \end{aligned}$$

OR $\underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$