<u>1-1 & onto</u> 23 Nov 09 (1)

eg 1) Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

a) Is A 1-1?

$$|A| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \cancel{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}}^{\,0} - \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = +1 \neq 0$$

So A is 1-1

b) Is A onto?

A is square & 1-1 so A is onto.

(1-1 & onto are ~~same thing~~ equivalent for square mxs)

c) Find T_A^{-1}

$T_A^{-1} = T_{A^{-1}}$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \quad R_3 \to R_3 - R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right) \quad R_2 \to R_2 - R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right) \quad R_1 \to R_1 - R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right) \quad A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$T_A^{-1}(\underline{x}) = T_{A^{-1}}(\underline{x}) = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\left(\underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \right)$

2) Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$ (2)

a) Is T_A onto?

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix} \quad R_3 \to R_3 - R_1$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad R_3 \to R_3 - R_2$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$ — Row of 0's hence not onto.

[Could have shown $\det(A) = 0$]

b) Is T_A 1-1?

REF has a col without a pivot (col 3)

So not ~~one~~ 1-1

OR $\text{rank}(A) = 2 \neq 3$ # of cols of A

So not 1-1.

3) Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 1 \end{pmatrix}$

a) Is T_A onto?

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 1 \end{pmatrix} \quad R_3 \to R_3 - 2R_1 \quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}$$

$$R_3 \to R_3 + R_2 \quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$ So not onto

b) is T_A 1-1?

Yes $\text{rank}(A) = 2 = \#$ cols of A.

c) T_A is not ont (by part A) ~~Find the~~ $\Rightarrow \text{Ran}(T_A) \neq \mathbb{R}^3$

Find $\text{Ran}(T_A)$

3

$$\left(\begin{array}{cc|c} 1 & 1 & x \\ 0 & 1 & y \\ 2 & 1 & z \end{array}\right) \quad R_3 \to R_3 - 2R_1 \quad \left(\begin{array}{cc|c} 1 & 1 & x \\ 0 & 1 & y \\ 0 & -1 & z-2x \end{array}\right)$$

$$R_3 \to R_3 + R_2 \quad \left(\begin{array}{cc|c} 1 & 1 & x \\ 0 & 1 & y \\ 0 & 0 & z-2x+y \end{array}\right)$$

So Image pts satisfy $z-2x+y=0$

So $\mathrm{Ran}(T_A) =$ Plane $2x-y-z=0$

3) Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

a) Is T_A 1-1?

<u>No.</u> $\mathrm{rank}(A) \leq$ # of ~~cols~~ rows so must be a col without a pivot
$= 2$

b) Is T_A onto?

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad R_2 \to R_2 - R_1 \quad \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So <u>Is Onto</u> (# of rows = rank(A))