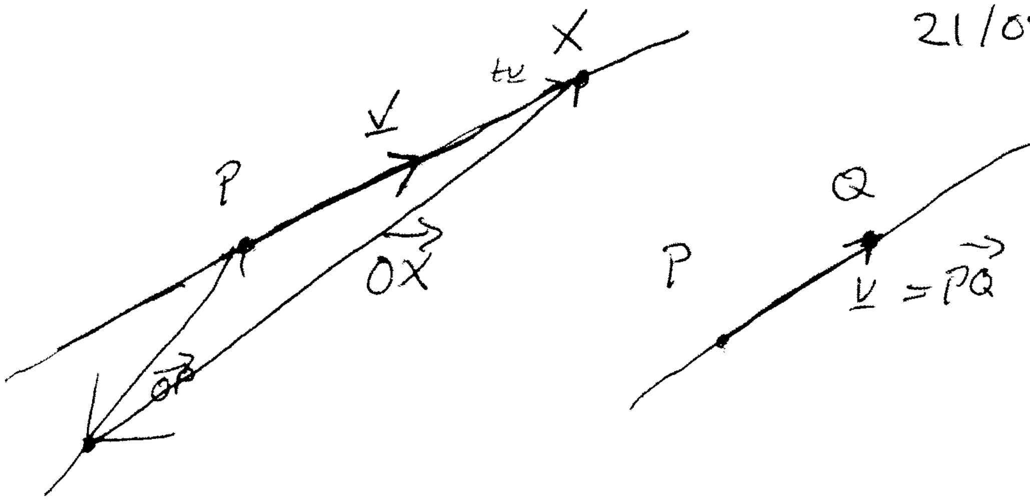
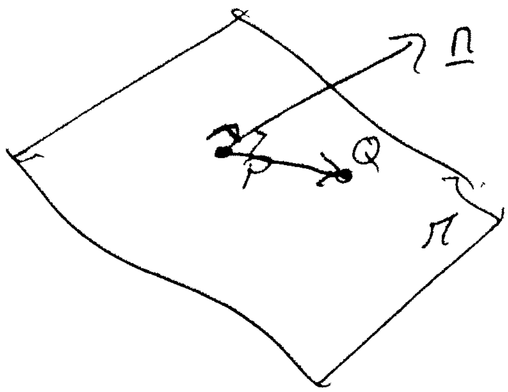
21/09/09



Note: $\overrightarrow{PQ}$ is $\parallel$ to the line.

$t\underline{v} = \overrightarrow{PX}$ (each t defines a new X)



$\overrightarrow{PQ} \perp \underline{n}$

$\therefore \overrightarrow{PQ} \cdot \underline{n} = 0$

$n = (a,b,c) \quad P = (x_0, y_0, z_0) \quad Q(x,y,z)$

$\underline{n} \cdot \overrightarrow{PQ} = a(x-x_0) + b(y-y_0) + c(z-z_0) = 0.$

$ax + by + cz = ax_0 + by_0 + cz_0$

Note $ax_0 + by_0 + cz_0 = \underline{n} \cdot \overrightarrow{OP} := d$

$\boxed{ax + by + cz = d}$

Where $d = \underline{n} \cdot \overline{OP}$

$n_1 + n_3 = 0$ ① | $x + z = 0$

$2n_1 - n_2 + 2n_3 = 0$ ② | $2x - y + 2z = 0$

① $\Rightarrow$ $n_1 = -n_3$ $\Rightarrow$ $2(n_1 + n_3) = 0$

② $\Rightarrow$ $-n_2 = 0$ $n_2 = 0$

Let $t \in \mathbb{R}$ set $t = n_1 = -n_3$

~~Ek~~ Any $\text{sol}^{\underline{n}}$ looks like $(t, 0, -t)$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad - \text{Line in } \mathbb{R}^3 \qquad n_1, n_2, n_3$$

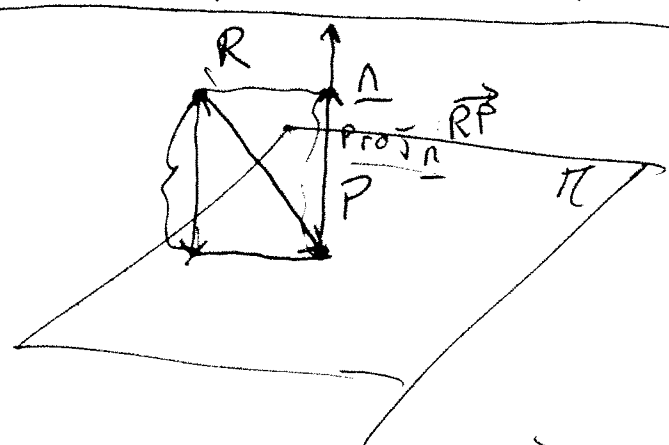
line $\perp$ to the plane Π

$-n_1 + n_3 = 0$ $\Rightarrow$ $n_3 = n_1 = 0$

$n_1 = 0$

Let $n_2 = t$ $(0, t, 0)$

Distance of a Pt to a ~~line~~ Plane



distance is $\|proj_{\underline{n}} \vec{PR}\| = comp_{\underline{n}} \vec{PR}$

$= \dfrac{|\underline{n} \cdot \vec{PR}|}{\|\underline{n}\|}$

Note: If $\|\underline{n}\| = 1$ then distance is $\underline{n} \cdot \vec{PR}$

If $R = O$ — distance to origin of plane $ax+by+cz=d$

then $\underline{n} \cdot \vec{OP} = d$

In General $\dfrac{d}{\|\underline{n}\|}$ is the distance of $ax+by+cz=d$ to the origin.

$n = (a, b, c)$

Given Plane $ax+by+cz=d$ & a Pt $R(x_1, y_1, z_1)$

Distance between then is $\boxed{\dfrac{|\underline{n} \cdot \vec{PR}|}{\|\underline{n}\|} = \dfrac{|ax_1 + by_1 + cz_1|}{\sqrt{a^2+b^2+c^2}}}$

eg Find the distance of $R = (1, 3, 3)$ to the plane

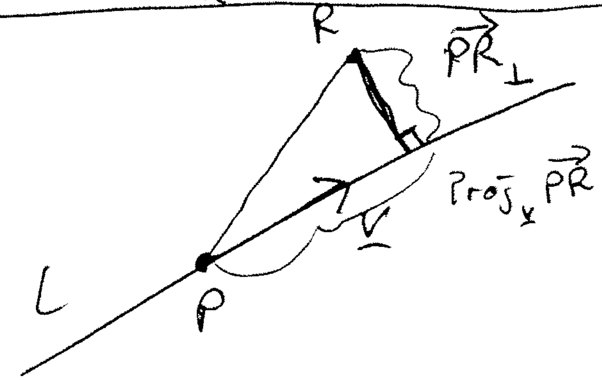
$3x + y - z = 5$

$\underline{n} = (3, 1, -1)$ $\quad \|\underline{n}\| = \sqrt{3^2+1^2+1^2} = \sqrt{11}$

Need a Pt on the Plane P. $(1, 2, 0)$ ← Picked to satisfy eq^n of Plane

$\dfrac{|\underline{n} \cdot \vec{PR}|}{\|\underline{n}\|} = \dfrac{|(3,1,-1) \cdot (0,1,3)|}{\sqrt{11}} = \dfrac{|1-3|}{\sqrt{11}} = \dfrac{2}{\sqrt{11}}$

Distance of a Pt to a line



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{OP} + t\underline{v}$$

$$\vec{PR}_\perp = PR - \mathrm{proj}_{\underline{v}} \vec{PR}$$

~~Given~~ Given $P = (x_0, y_0, z_0)$ $\quad R = (x, y, z)$

$v = (a, b, c)$ $\quad$ (// to line)

Distance from R to L is $\|\vec{PR}_\perp\|$

$= \|\vec{PR} - \mathrm{proj}_{\underline{v}} \vec{PR}\|$

<u>eg</u> Given L: $\underline{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ find the

distance to $R = (2, 1, 2)$

$P = (1, 1, 1)$ $\qquad \underline{v} = (1, 2, 0)$

$\vec{PR} = (1, 0, 1)$

$\vec{PR} \cdot \underline{v} = 1$ $\qquad \|v\|^2 = 5$

$\mathrm{proj}_{\underline{v}} \vec{PR} = \dfrac{\underline{v} \cdot \vec{PR}}{\|\underline{v}\|^2} \underline{v} = \frac{1}{5}(1, 2, 0)$

$\vec{PR}_\perp = \vec{PR} - \mathrm{proj}_{\underline{v}} \vec{PR} = (1, 0, 1) - \frac{1}{5}(1, 2, 0)$

$= \frac{1}{5}(5-1, 0-2, 5-0) = \frac{1}{5}(4, -2, 5)$

$\|\vec{PR}_\perp\| = \|\frac{1}{5}(4, -2, 5)\| = \dfrac{\sqrt{16+4+25}}{5} = \dfrac{\sqrt{45}}{5} = \dfrac{3\sqrt{5}}{5}$

Find the Pt. of intersect^n of the lines

eg Find the pt of intersect^n of

$L_1: \underline{x} = \begin{pmatrix}1\\1\\1\end{pmatrix} + t\begin{pmatrix}1\\2\\0\end{pmatrix}$ $\quad L_2: \underline{x} = \begin{pmatrix}2\\1\\2\end{pmatrix} + s\begin{pmatrix}4\\-2\\5\end{pmatrix}$

3 eq^n's in 2 unknowns

$$\begin{aligned} 1+t &= 2+4s \\ 1+2t &= 1-2s \\ 1 &= 2+5s \end{aligned}$$

$s = -\frac{1}{5} \quad \Rightarrow t = +\frac{1}{5}$

So intersect when $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6/5\\7/5\\1\end{pmatrix}$

(Sub $t = \frac{1}{5}$ into l_1)