Oct 19 (1)

WARNING **Never** **Ever** write $\frac{1}{A}$ ✗

for a matrix

Always use A^{-1} ✓

Can't make sense of $\frac{B}{A}$?? $A^{-1}B$ or BA^{-1}

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 5 & 8 \end{pmatrix} \qquad A^{-1} = \frac{1}{2}\begin{pmatrix} -15 & 1 & 5 \\ 1 & 1 & -1 \\ 5 & -1 & -1 \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Then $A\underline{x} = \underline{b}$:

$$\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 5y + 5z &= 2 \\ 3x + 5y + 8z &= 3 \end{aligned}$$

Sol^n $\underline{x} = A^{-1}\underline{b}$

$$\frac{1}{2}\begin{pmatrix} -15 & 1 & 5 \\ 1 & 1 & -1 \\ 5 & -1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Can also interpret this as saying T_A maps $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ to $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ i.e. $T_A(\underline{x}) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$T_{A^{-1}}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

eg 1) Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ Find A^{-1}

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \quad R_3 \to R_3 - R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array}\right) \quad R_2 \leftrightarrow R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 2 & 1 & 0 & 1 & 0 \end{array}\right) \quad R_3 \to R_3 - 2R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 & -2 \end{array}\right) \quad R_1 \to R_1 - R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 & -2 \end{array}\right)$$

$$\underbrace{}_{I} \qquad \underbrace{}_{A^{-1}}$$

$$A^{-1} = \begin{pmatrix} -1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{pmatrix}$$

Check: $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$ ✓

2) If $A = \begin{pmatrix} 2 & 10 & 10 \\ 2 & 9 & 9 \\ 2 & 9 & 10 \end{pmatrix}$ Find A^{-1}

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$$\left(\begin{array}{ccc|ccc} 2 & 10 & 10 & 1 & 0 & 0 \\ 2 & 9 & 9 & 0 & 1 & 0 \\ 2 & 9 & 10 & 0 & 0 & 1 \end{array}\right)$$

~~$R_1 \to \frac{1}{2}R$~~
$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

~~$1 \; 5 \; 5 \; \frac{1}{2} \; 0 \; 0$~~

$$\left(\begin{array}{ccc|ccc} 2 & 10 & 10 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{array}\right) \quad R_2 \leftrightarrow R_3$$

$$\left(\begin{array}{ccc|ccc} 2 & 10 & 10 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & -1 & -1 & -1 & 1 & 0 \end{array}\right) \quad R_2 \to -R_2$$

$$\left(\begin{array}{ccc|ccc} 2 & 10 & 10 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & -1 & -1 & 0 & 1 & -1 \end{array}\right) \quad R_3 \to R_3 + R_2$$

$$\left(\begin{array}{ccc|ccc} 2 & 10 & 10 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array}\right) \quad R_3 \to -R_3$$

$$\left(\begin{array}{ccc|ccc} 2 & 10 & 10 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right) \quad R_1 \to R_1 - 10R_3$$

$$\left(\begin{array}{ccc|ccc} 2 & 10 & 0 & 1 & 10 & -10 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right) \quad R_1 \to R_1 - 10R_2$$

$$\left(\begin{array}{ccc|ccc} 2 & 0 & 0 & -9 & 10 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right) \quad R_1 \to \frac{1}{2}R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{9}{2} & 5 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right)$$

So $A^{-1} = \begin{pmatrix} -\frac{9}{2} & 5 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$

<u>Check:</u> $\begin{pmatrix} 2 & 10 & 10 \\ 2 & 9 & 9 \\ 2 & 9 & 10 \end{pmatrix} \begin{pmatrix} \frac{9}{2} & 5 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix} = I$ ✓

b) Use A^{-1} above to solve

$$\begin{aligned} 2x + 10y + 10z &= 2 \\ 2x + 9y + 9z &= 2 \\ 2x + 9y + 10z &= 3 \end{aligned}$$

$A\underline{x} = \underline{b}$ $\quad \underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\quad A = \begin{pmatrix} 2 & 10 & 10 \\ 2 & 9 & 9 \\ 2 & 9 & 10 \end{pmatrix}$ $\quad \underline{b} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$

$\underline{x} = A^{-1}\underline{b}$ $\quad \begin{pmatrix} -\frac{9}{2} & 5 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$x = 1, \quad y = -1, \quad z = 1$

c) As above but $\underline{b} = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}$

$$\begin{pmatrix} -\frac{9}{2} & 5 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 16 \\ -1 \\ -2 \end{pmatrix}$$

$x = 16, \quad y = -1, \quad z = -2.$

3) Find A^{-1} where $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 3 & 3 \end{pmatrix}$ 5

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 3 & 0 & 0 & 1 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & -1 & -2 & 0 & 1 \end{array}\right) \quad R_3 \to R_3 - R_2$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{array}\right)$$

A^{-1} DNE

$A\underline{x} = b$ has eith ∞ sol^{n}s or no sol^{n}s depending on $\underline{b}$

Last row says $0 = -b_1 - b_2 + b_3$

So sol^{n}s iff. $b_1 + b_2 - b_3 = 0$ — 1 parameter family.