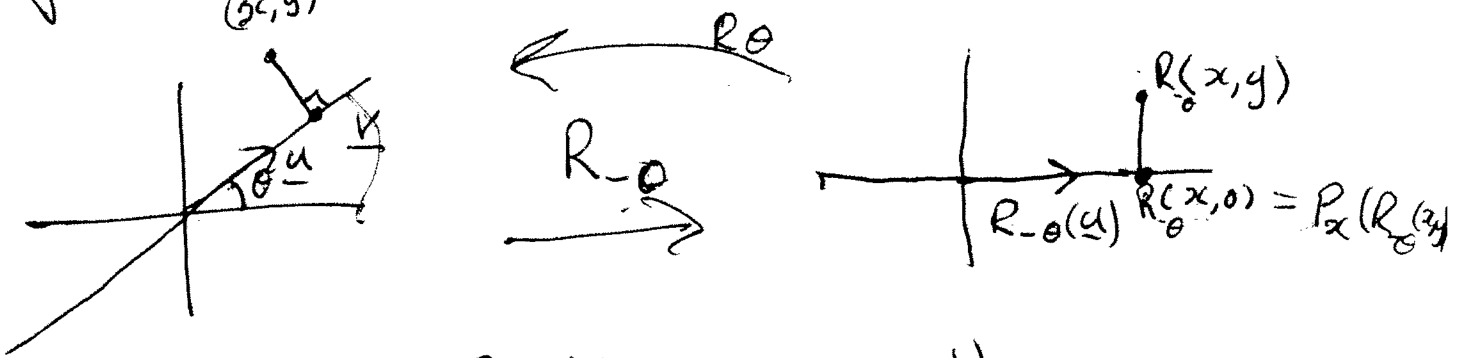
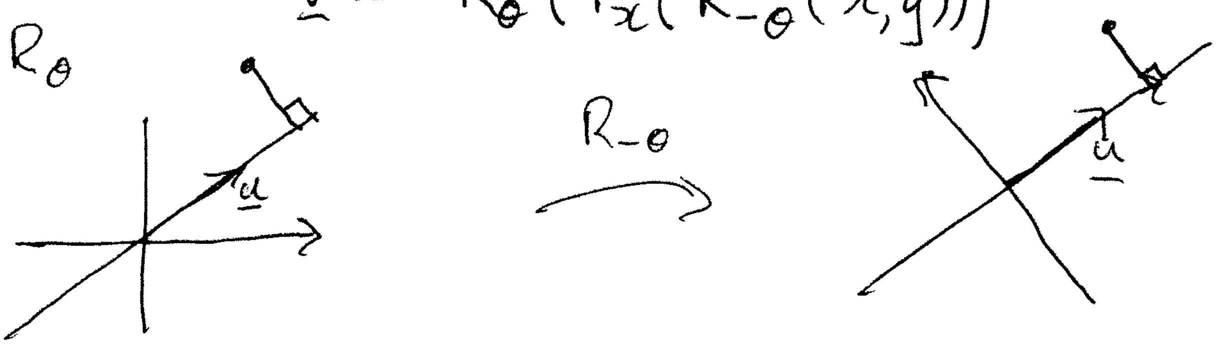
18 Nov 09 (1)

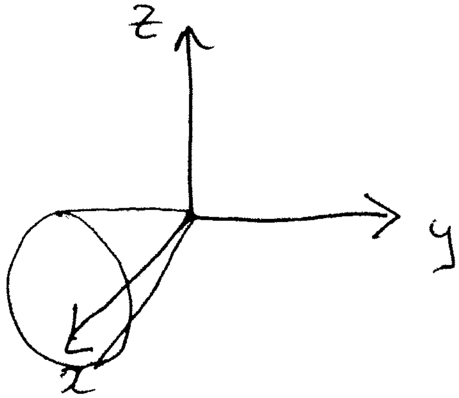
Projectⁿ onto x-axis in $\mathbb{R}^2$



$$\underline{v} = R_\theta(P_x(R_{-\theta}(x,y)))$$

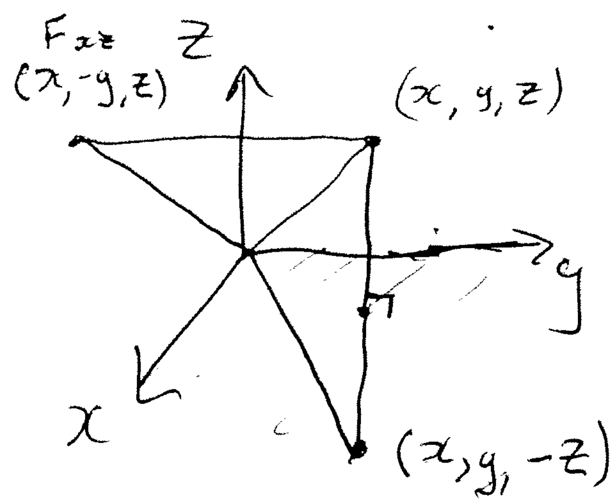


Rotatⁿ about x-axis in $\mathbb{R}^3$



x coord. doesn't change
rotatⁿ in the yz-plane.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \multicolumn{2}{c}{\multirow{2}{*}{R_θ}} \\ 0 & & \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ - \\ - \end{pmatrix}$$



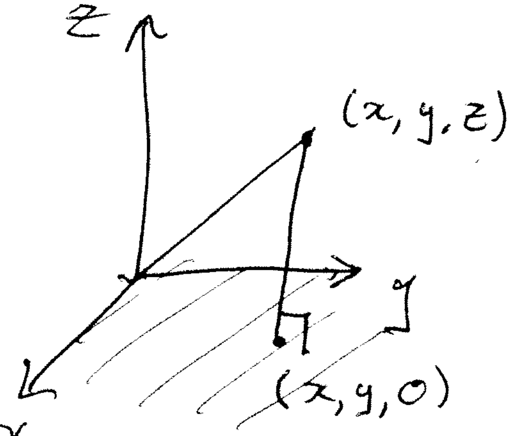
$F_{xy}(1,0,0) = (1,0,0)$

$F_{xy}(0,1,0) = (0,1,0)$

$F_{xy}(0,0,1) = (0,0,-1)$

$$[F_{xy}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Project^{n}s in $\mathbb{R}^3$



P_{xy}

$P_{xy}(1,0,0) = (1,0,0)$
$P_{xy}(0,1,0) = (0,1,0)$
$P_{xy}(0,0,1) = (0,0,0)$

$$[P_{xy}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Composition of Maps

We know that composit^{n} of matrix maps is just matrix mult^{n} i.e. $T_A(T_B(\underline{x})) = T_{AB}(\underline{x})$

eg 1) Find the standard mx for the transformat^{n} in $\mathbb{R}^2$ given by a reflect^{n} about the x-axis, followed by a rotat^{n} by $60°$ ~~counter~~ counter clockwise.

$$F_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad \theta = 60° = \frac{\pi}{3}$$

$$R_{\frac{\pi}{3}} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

$$T_A = R_\theta(F_x(\underline{x})) \qquad A = [R_\theta][F_x] -$$

$$= \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

2) Find the ~~std~~ the std mx for the transformat$^{\text{n}}$ in $\mathbb{R}^3$ obtained by a counterclockwise rotat$^{\text{n}}$ about the y-axis of $60°$ followed by a dilat$^{\text{n}}$ by a factor 2. ③

$$[R_\theta] = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \qquad [D_2] = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$A = [D_2][R_{\frac{\pi}{3}}] = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 & +\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & +\sqrt{3} \\ 0 & 2 & 0 \\ -\sqrt{3} & 0 & 1 \end{pmatrix}$$

3) Find the standard mx for the mapping in $\mathbb{R}^3$ obtained by a reflect$^{\text{n}}$ about the yz-plane, followed by an orthogonal project$^{\text{n}}$ onto the xz-plane.

$$[R_{y,z}] = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad [P_{x,z}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$[P_{xz}][R_{yz}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Subspaces — Review

For all
1) $\forall\, \underline{x}, \underline{y} \in U \quad \underline{x}+\underline{y} \in U$
2) $\forall\, \underline{x} \in U, c \in \mathbb{R} \quad c\underline{x} \in U$

eg 1) $U = \{ (x, x^2) \mid x \in \mathbb{R} \}$

the set of   form   such that   condit^n

[Not a Subspace]

[Look for a pair of vectors where sum rule fails]

$(1,1) + (2,4) = (3,5) \qquad 5 \neq 3^2$

So $(1,1) \in U$, $(2,4) \in U$ but $(1,1)+(2,4) \notin U$.

- So Rule 1 does not hold for every pair.

Alternately could have shown Rule 2 broken.

$2(1,1) = (2,2) \notin U \quad (2^2 \neq 2)$

2) $U = \cancel{\{ \underline{x}+\underline{a} \in \mathbb{R}^2 \mid \underline{a}=(1,1), \underline{x} \in \mathbb{R}^2 \}}$

$\{ (x, \overset{y}{\cancel{x+1}}) \cancel{\in U} \mid x, y \in \mathbb{R},\ x, y > 0 \}$

$(0,0) \notin U$ so not a subspace.

3) $U = \{ \overset{(x,y,z)}{\underline{x}} \in \mathbb{R}^3 \mid ax+by+cz=0,\ a,b,c \text{ fixed} \}$

(the Plane $ax+by+cz=0$ through.)

The solut^n to any homogeneous system of eq^{n}s is always a subspace

$ax+by+cz=0$ is 1 homogeneous eq^n in 3 unknowns.

$U = \{(x,y,z) \in \mathbb{R}^3 \mid ax+by+cz=0\}$

Let ~~u x~~ $\underline{u}$ & $\underline{v} \in U$.

So $au_1 + bu_2 + cu_3 = 0$ & $av_1 + bv_2 + cv_3 = 0$

[- Defining Property of vectors in U.]

[Must Show $\underline{u}+\underline{v}$ has defining property of U so $\underline{u}+\underline{v} \in U$]

Consider $\underline{u}+\underline{v} = (u_1+v_1, u_2+v_2, u_3+v_3)$

Consider $a(u_1+v_1) + b(u_2+v_2) + c(u_3+v_3)$ [Have to show this is 0]

$$= \underbrace{au_1 + bu_2 + cu_3}_{0} + \underbrace{av_1 + av_2 + av_3}_{0}$$

$$= 0 + 0 = 0$$

So $\underline{u}+\underline{v} \in U$

Let $\underline{u} \in U$ & $d \in \mathbb{R}$ so $au_1 + bu_2 + cu_3 = 0$

$d\underline{u} = (du_1, du_2, du_3)$

Consider $a(du_1) + b(du_2) + c(du_3)$

$= 0$

So $d\underline{u} \in U$.

4) Show $\{(x,y,z) \in \mathbb{R}^3 \mid ax+by+cz=1\}$ is NOT a subspace.

$a0 + b0 + c0 = 0 \neq 1$

So $(0,0,0) \notin U$. $\Rightarrow$ Not a s.s.

$\underset{\sim}{u}_i \in \mathbb{R}^n$

Lin indep $\quad a_1 \underset{\sim}{u}_1 + a_2 \underset{\sim}{u}_2 + \dots + a_k \underset{\sim}{u}_k = \underset{\sim}{0}$

has only the trivial solⁿ $a_1 = a_2 = \dots = a_k = 0$

Method 1 $\quad (k \neq n)$ Form the matrix whose cols are the $\underset{\sim}{u}_i$ $\quad A_u = (\underset{\sim}{u}_1 | \underset{\sim}{u}_2 | \dots | \underset{\sim}{u}_k)$

$A_u \underset{\sim}{a} = \underset{\sim}{0}$

$\underset{\sim}{u}_1 = (1,0,1) \quad \underset{\sim}{u}_2 = (1,1,0) \quad k=2,\ n=3$

$$a_1 \begin{pmatrix}1\\0\\1\end{pmatrix} + a_2 \begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}1&1\\0&1\\1&0\end{pmatrix}\begin{pmatrix}a_1\\a_2\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\left(\begin{array}{cc|c}1&1&0\\0&1&0\\1&0&0\end{array}\right) \quad R_3 \to R_3 - R_1$$

$$\left(\begin{array}{cc|c}1&1&0\\0&1&0\\0&-1&0\end{array}\right) \quad R_3 \to R_3 + R_2$$

$$\left(\begin{array}{cc|c}1&1&0\\0&1&0\\0&0&0\end{array}\right)$$

only solⁿ $a_1 = a_2 = 0$

So lin indep

Method 2 only works when $k = n$

Now A_u is square

Homogeneous system has non-trivial solⁿ iff $\det(A_u) \neq 0$

eg $\quad \underset{\sim}{u}_1 = (1,0,1) \quad \underset{\sim}{u}_2 = (1,1,0) \quad \underset{\sim}{u}_3 = (2,1,1)$

$$|A_u| = \begin{vmatrix}1&1&2\\0&1&1\\1&0&1\end{vmatrix} = \begin{vmatrix}1&1\\0&1\end{vmatrix} - \begin{vmatrix}0&1\\1&1\end{vmatrix} + 2\begin{vmatrix}0&1\\1&0\end{vmatrix}$$

$$= 1 + 1 - 2 = 0$$

So lin. dep. In fact $\underset{\sim}{u}_1 + \underset{\sim}{u}_2 - \underset{\sim}{u}_3 = \underset{\sim}{0}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) \quad R_3 \to R_3 - R_1$$

$$\begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{pmatrix} \quad R_3 \to R_3 - R_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

∞ many sol^{n}s.
So Dependent.

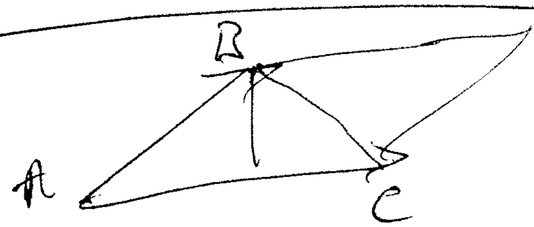
Let $t \in \mathbb{R}$, $z = t$, $y = -t$, $x = t - 2t = -t$

$t(-1, -1, 1)$

$t\underline{u}_1 + t\underline{u}_2 - tu_3 = \underline{0}$ for any $t \in \mathbb{R}$.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right) \qquad \Big(\begin{matrix} 1 & 1 & 1 & 0 \\ 0 & & & \end{matrix}$$



$$\frac{\frac{1}{2} \| \overrightarrow{AB} \times \overrightarrow{AC} \|}{\frac{1}{2} \underset{\overrightarrow{AC}}{\text{base}} \times \text{height}}$$