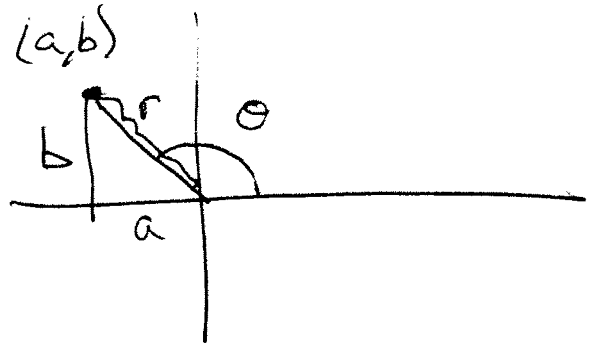
MTH 141
16 Sept. 09

Unit Vectors in $\mathbb{R}^2$

$\mathbb{R}^2$

$\perp$
$u \cdot v = 0$



$(a,b) = r(\cos\theta, \sin\theta)$

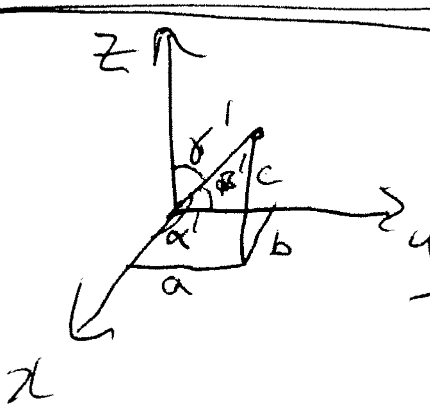
In polar coordinates the vector / point (a,b) is represented by (r, θ)

$a = r\cos\theta$
$b = r\sin\theta$

$r = \|(a,b)\| = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$ (See caveats about $\tan^{-1}$)

Direct^n cosines in $\mathbb{R}^3$



$\underline{v} = (a,b,c)$

$\alpha = \cos\alpha' = \frac{a}{\|v\|}$

$\beta = \cos\beta' = \frac{b}{\|v\|}$

$\gamma = \cos\gamma' = \frac{c}{\|v\|}$

} direct^n cosines of $\underline{v}$

Given v $\frac{1}{\|v\|}\underline{v} = \left(\frac{a}{\|v\|}, \frac{b}{\|v\|}, \frac{c}{\|v\|}\right) = (\alpha, \beta, \gamma)$
is the unit vector $\parallel$ to $\underline{v}$

eg 1) $\underline{u} = (1,0,1,0)$ $\quad \underline{v} = (1,1,1,0)$

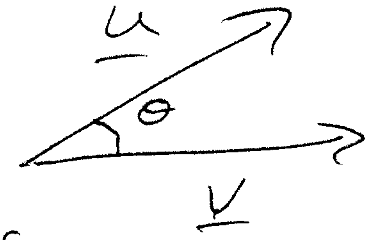
Find the angle between $\underline{u}$ & $\underline{v}$.

$\underline{u} \cdot \underline{v} = (1,0,1,0) \cdot (1,1,1,0) = 2$

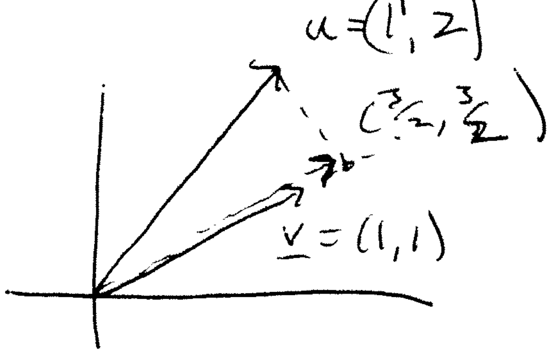
$\|u\| = \sqrt{2} \qquad \|v\| = \sqrt{3}$

$\cos\theta = \dfrac{u \cdot v}{\|\underline{u}\|\|\underline{v}\|} = \dfrac{2}{\sqrt{2}\sqrt{3}} = \dfrac{2}{\sqrt{6}}$

$\theta = \cos^{-1}\dfrac{2}{\sqrt{6}}$



Note any two vectors in $\mathbb{R}^n$ define a plane.

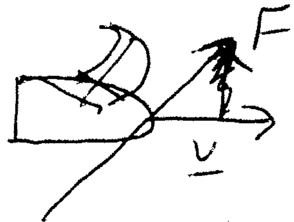


$\|(\tfrac{3}{2}, \tfrac{3}{2})\|$

$= \sqrt{\left(\dfrac{3^2}{2^2} + \dfrac{3^2}{2^2}\right)} = \sqrt{\dfrac{18}{4}} = \dfrac{\sqrt{18}}{2}$

$= 3\dfrac{\sqrt{2}}{2} = \dfrac{3}{\sqrt{2}} \neq 1$

eg 1)



How much of $\underline{F}$ is pushing the sailing boat along?

that is exactly $\text{comp}_{\underline{v}}\underline{F}$

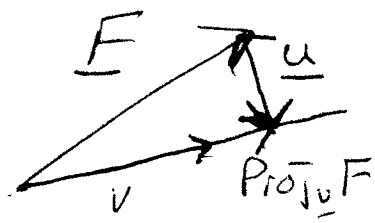
Take $\underline{v} = (1,0)$

$\underline{F} = (10^5, 10^5)$

$\underline{v} \cdot \underline{F} = 10^5 + 0 = 10^5$

$\|\underline{v}\| = 1$

$$\boxed{\text{comp}_{\underline{v}}\underline{F} = \frac{10^5}{1}}$$



$\text{proj}_{\underline{v}}\underline{F} = 10^5 (1,0)$

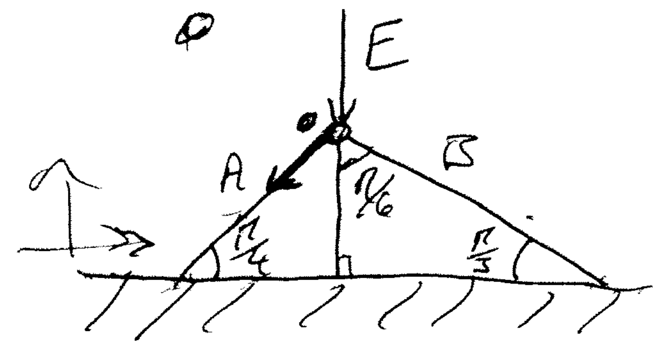
$\underline{u} = \underline{F} - \text{proj}_{\underline{v}}\underline{F}$

$\underline{u} = (10^5, 10^5) - (10^5, 0)$

$= (0, 10^5)$

$\|\underline{u}\| = 10^5$

eg: If a force of ~~10^5~~ 4×10^5 N. is applied directly down on the pivot

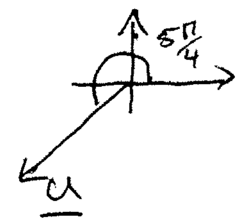


a) What is the force along each of the beams A & B

$\underline{F} = (0, -4 \times 10^5)$

Need a unit vector along each of A & B.

A:



$\underline{u} = (\cos\theta, \sin\theta) = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

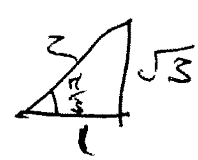
$\underline{u} \cdot \underline{F} = \frac{4}{\sqrt{2}} \times 10^5$

~~$\text{Proj}_{\underline{u}} \underline{F}$~~ $\text{comp}_{\underline{u}} \underline{F} = \frac{\underline{u} \cdot \underline{F}}{\|\underline{u}\|} = 2\sqrt{2} \times 10^5$

B:

$v = \left(\cos\left(-\frac{\pi}{3}\right), \sin\left(-\frac{\pi}{3}\right)\right)$

$= \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$



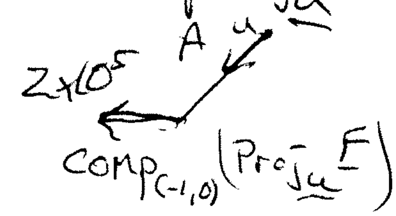
$\underline{F} \cdot \underline{v} = +\frac{\sqrt{3}}{2} \cdot 4 \times 10^5 = 2\sqrt{3} \times 10^5$

$\text{Comp}_{\underline{v}} \underline{F} = \frac{u \cdot v}{\|u\|} = 2\sqrt{3} \times 10^5$.

b) What is the lateral force at the base of the Beam A.

= Component of $\underline{F}$ in dir (-1,0) of $\text{proj}_{\underline{u}} F$

$\text{Proj}_{\underline{u}} \underline{F} = \left(2\sqrt{2} \times 10^5\right)\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$



$(-1, 0) \cdot (2\sqrt{2} \times 10^5)\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 2 \times 10^5$

$\text{COMP}_{(-1,0)}\left(\text{Proj}_{\underline{u}} \underline{F}\right) = 2 \times 10^5$ N.