Preface

The intersection of graph searching and probabilistic methods is a new topic within graph theory, with applications to graph searching problems such as the game of Cops and Robbers and its many variants, graph cleaning, Firefighting, and acquaintance time. Research on this topic emerged only over the last few years, and as such, it represents a rapidly evolving and dynamic area. Before we give a definition of this topic, we give some background on three of its key constituents: the probabilistic method, random graphs, and graph searching.

The *probabilistic method* is a powerful nonconstructive tool in mathematics. While it has found tremendous success in combinatorics and graph theory, it has been successfully applied to many other areas of mathematics (such as number theory, algebra, and analysis) as well as theoretical computer science (for example, randomized algorithms). As one of its goals, the method may prove the existence of an object with given properties without actually finding it. A *random graph* is a graph that is generated by some random process. Although technically a topic within the probabilistic method, random graphs are an important topic in their own right. The theory of random graphs lies at the intersection between graph theory and probability theory, and studies the properties of typical random graphs. Random graphs have also found a natural home in the study of real-world complex networks such as the web graph and on-line social networks.

Graph searching deals with the analysis of games and graph processes that model some form of intrusion in a network, and efforts to eliminate or contain that intrusion. For example, in Cops and Robbers, a robber is loose on the network, and a set of cops attempts to capture the robber. How the players move and the rules of capture depend on which variant is studied. In graph cleaning, the network begins as contaminated, and brushes move between vertices and along edges to clean them. There are many variants of graph searching studied in the literature, which are either motivated by problems in practice, or are inspired by foundational issues in computer science, discrete mathematics, and artificial intelligence, such as robotics, counterterrorism, and network security. In the past few years, a number of problems have emerged from applications related to the structure of real-world networks that are expected to be large-scale and dynamic, and where agents can be probabilistic, decentralized, and even selfish or antagonistic. This is one of the reasons why the field of graph searching is nowadays rapidly expanding. Several new models, problems, or approaches have appeared, relating it to diverse fields such as random walks, game theory, logic, probabilistic analysis, complex networks, mobile robotics, and distributed computing.

Graph searching games and probabilistic methods take two separate, but intertwined approaches: the study of graph searching games on random graphs and processes, and the use of the probabilistic method to prove results about deterministic graphs. We will see both approaches many times throughout the book. One of goals of this monograph is to bring the intersection of probabilistic methods and graph searching games into a place more readily visible to researchers. While we do not claim to make an exhaustive account, the material presented here is a survey of some main results in this new field. Our intended audience is broad, including both mathematicians and computer scientists. Since our approach is to be self-contained wherever possible, much of the material is accessible to students (mainly graduate but also advanced undergraduates) with some background in graph theory and probability.

We present eleven chapters that can be read in order, or the first three chapters read first, with the reader then moving to whichever chapter captures their interest. Chapter 1 supplies the required background and notation in graph theory, asymptotics, and discrete probability used throughout the remaining chapters. Chapter 2 focuses on one of the most popular graph searching games, Cops and Robbers. We discuss there the cop number of random graphs, properties of almost all cop-win and k-cop-win graphs. Variants of Cops and Robbers, where for example, the players play on edges or the robber can move at infinite speed, are considered in Chapter 3. We devote Chapter 4 to the new vertex pursuit game of Zombies and Survivors, where the zombies (the cops) appear randomly and always move along shortest paths to the survivor (the robber). In Chapter 5, we discuss one of the most important conjectures in the area, Meyniel's conjecture on the cop number. We summarize there some recent work on the proof of the conjecture for random graphs. Chapter 6 focuses on graph cleaning, and Chapter 7 discusses acquaintance time. Chapter 8 focuses the Firefighter graph process, and Chapter 9 focuses on acquisition number. In Chapter 10, we present topics on temporal parameters such as capture time. The final chapter presents a number of miscellaneous topics, ranging from Revolutionaries and Spies, robot crawler, and toppling number, to the game of Seepage played on acyclic directed graphs.

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