Surrounding an Active Robber

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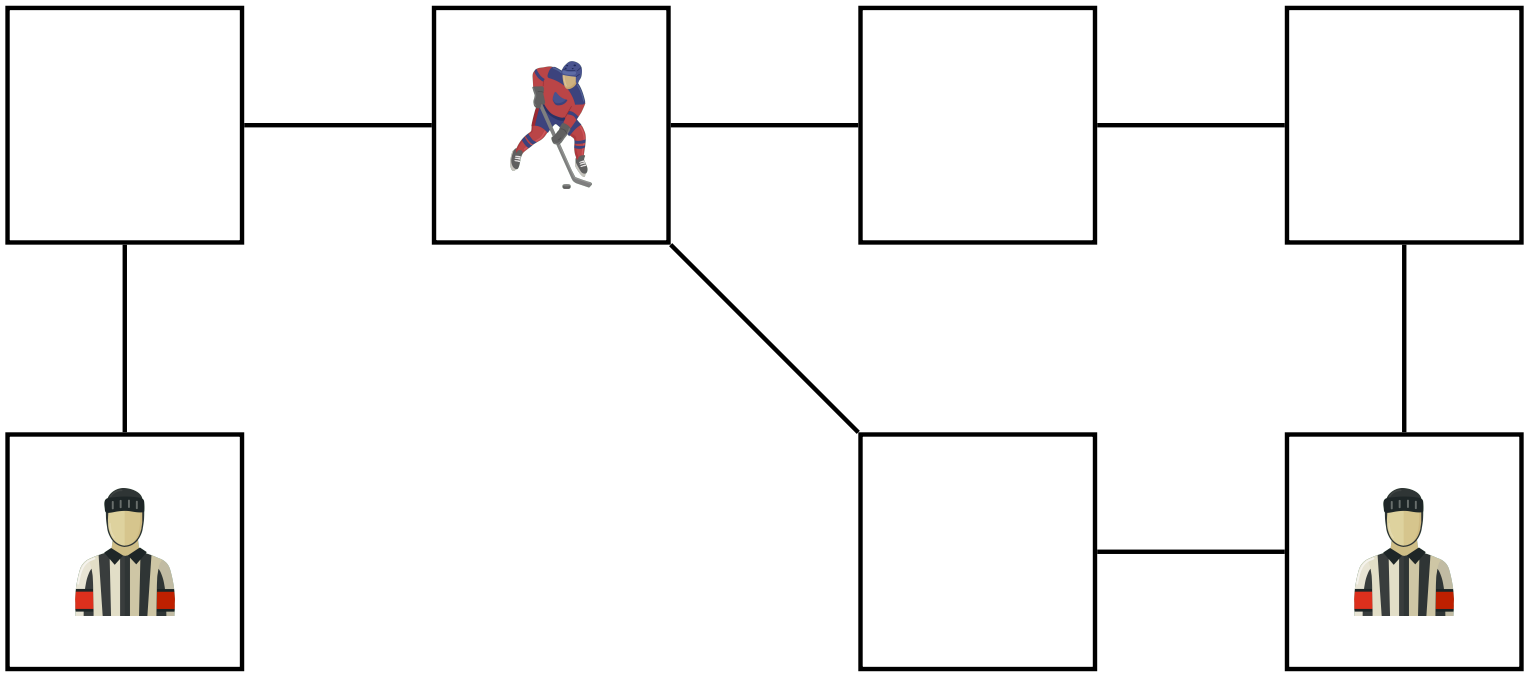
Joint Work with Nancy Clarke and Stephen Finbow

University of Prince Edward Island

GRASCan 2022
August 4, 2022

Cops and Robber



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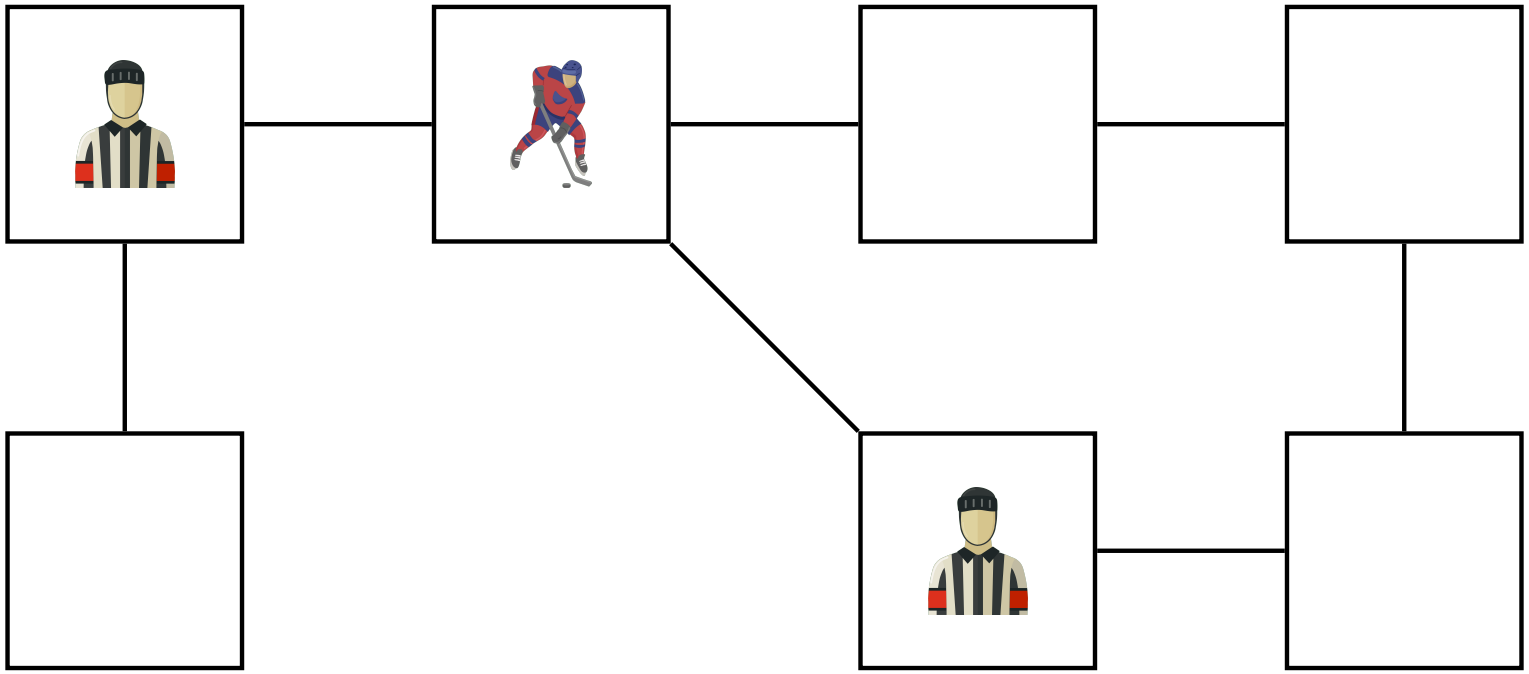

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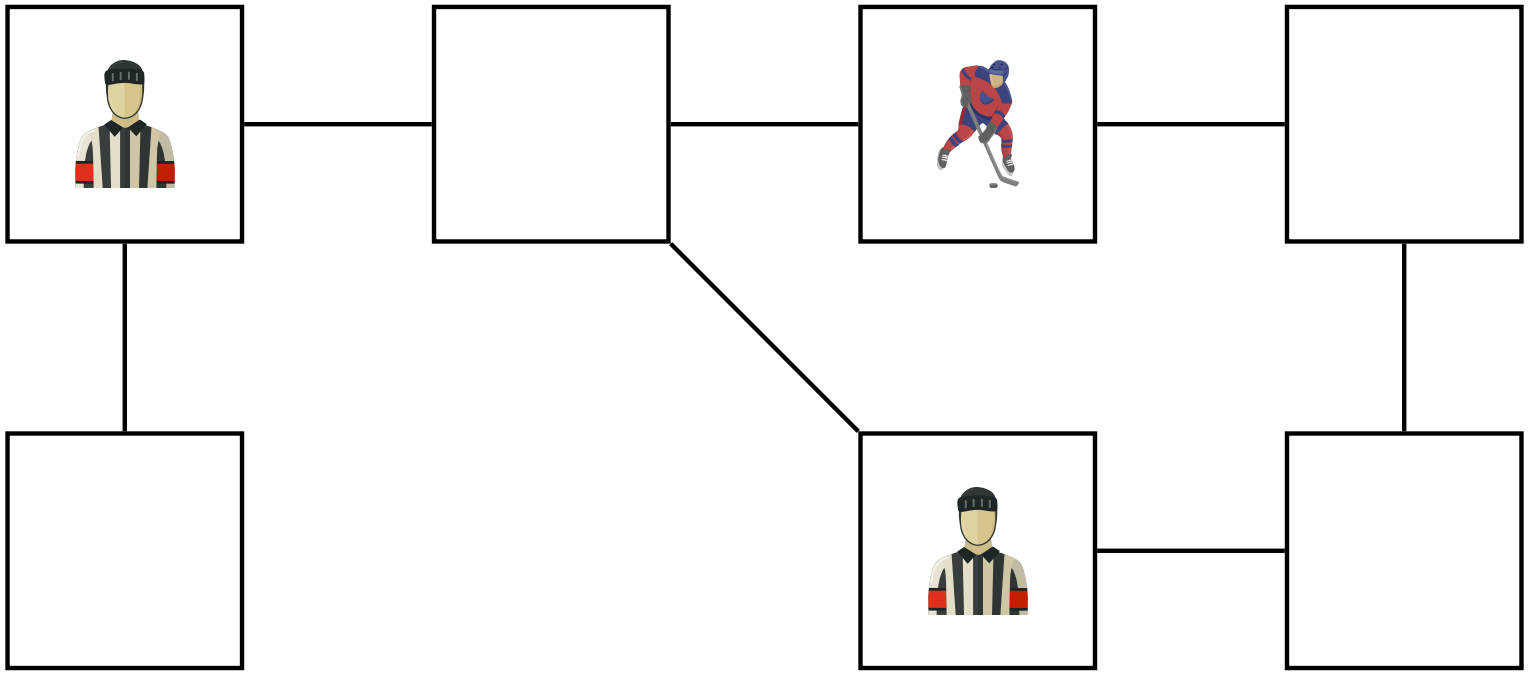

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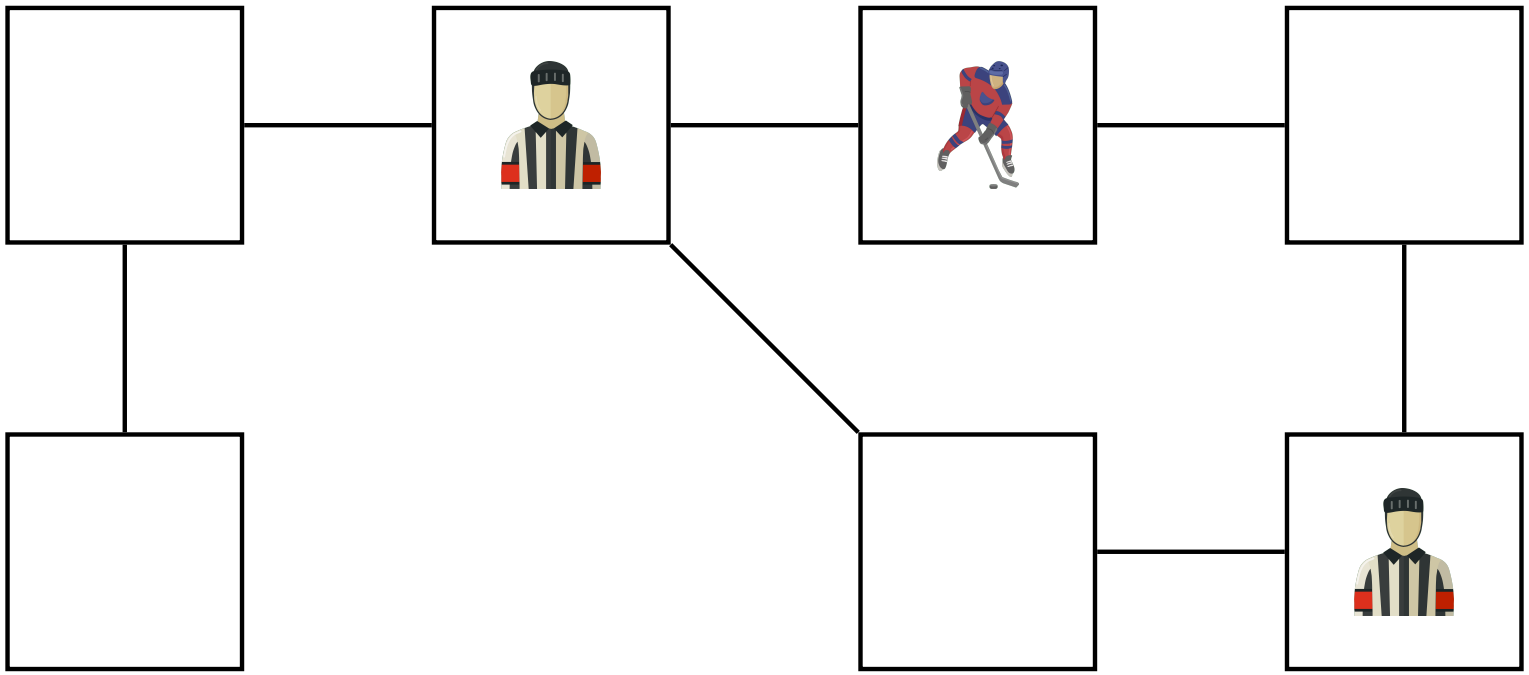

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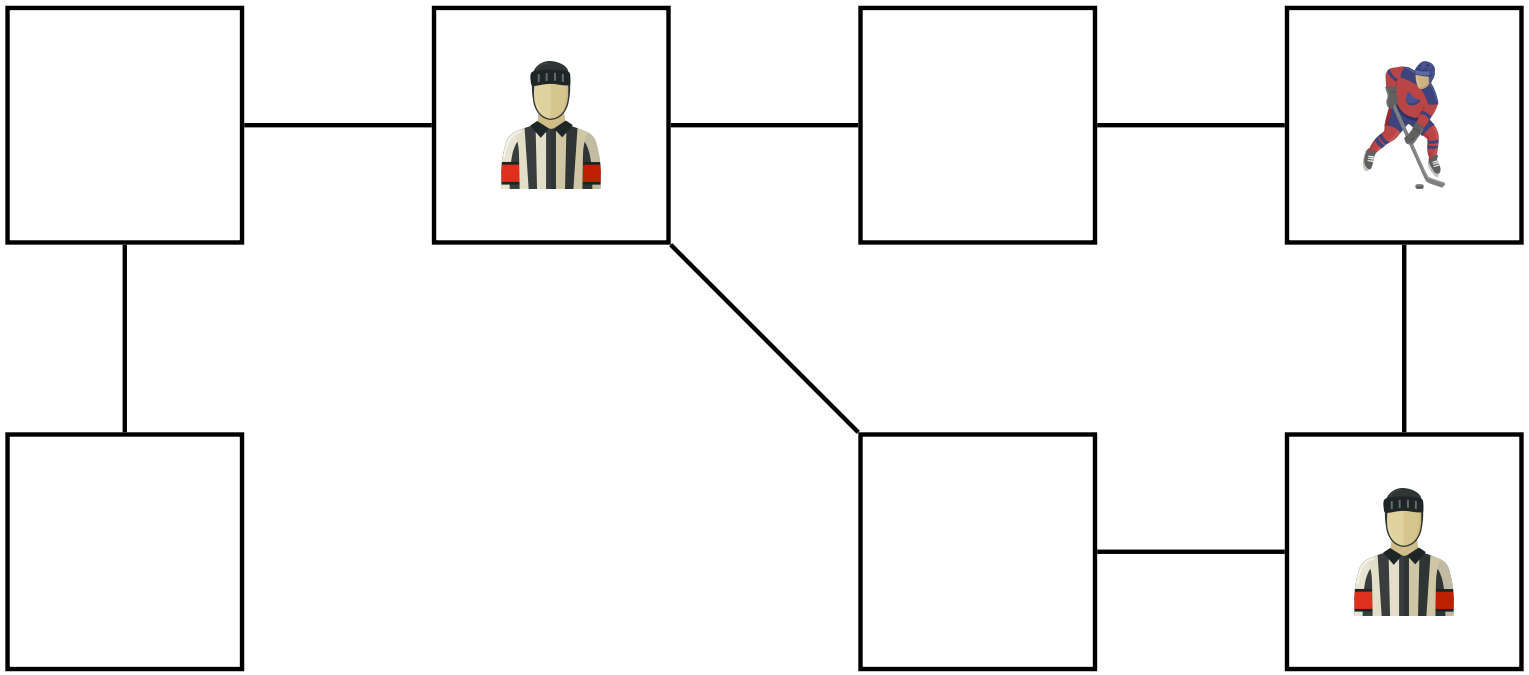

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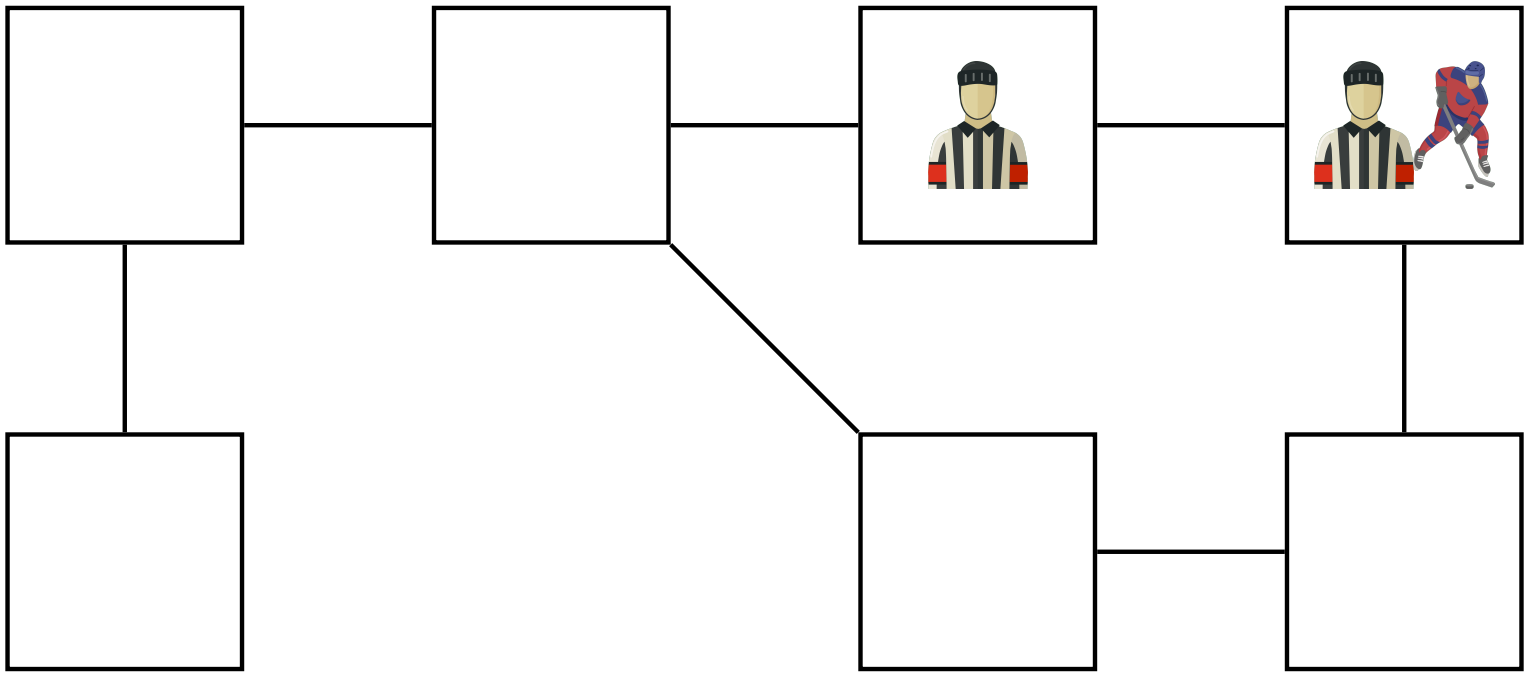

Surrounding a Robber

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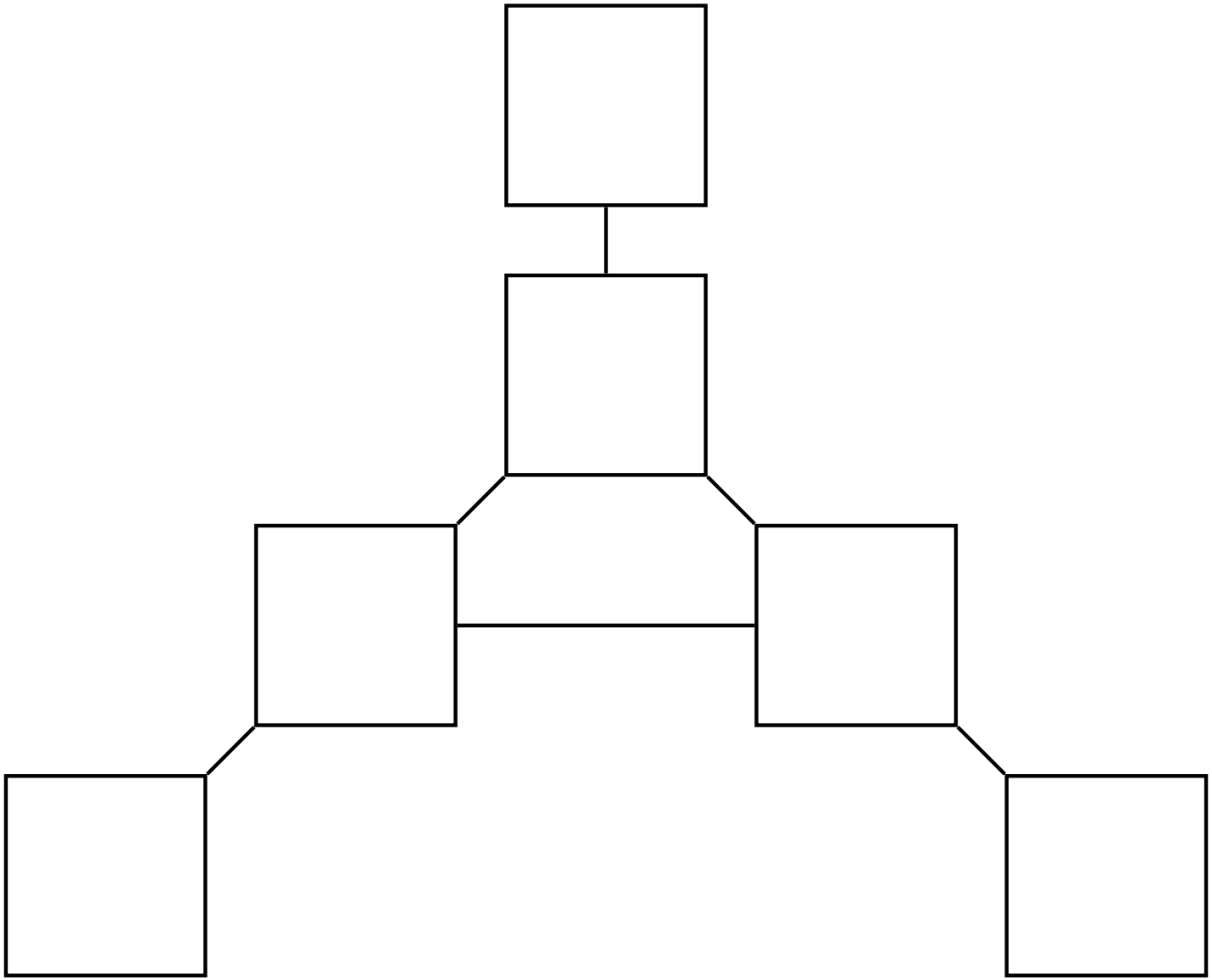
- A Cop landing on the Robber's vertex does not end the game, but rather simply compels him to move on his next turn
- The Robber may never move to any Cop's vertex
- The Robber only loses when every one of his adjacent vertices contains a Cop (he is surrounded)



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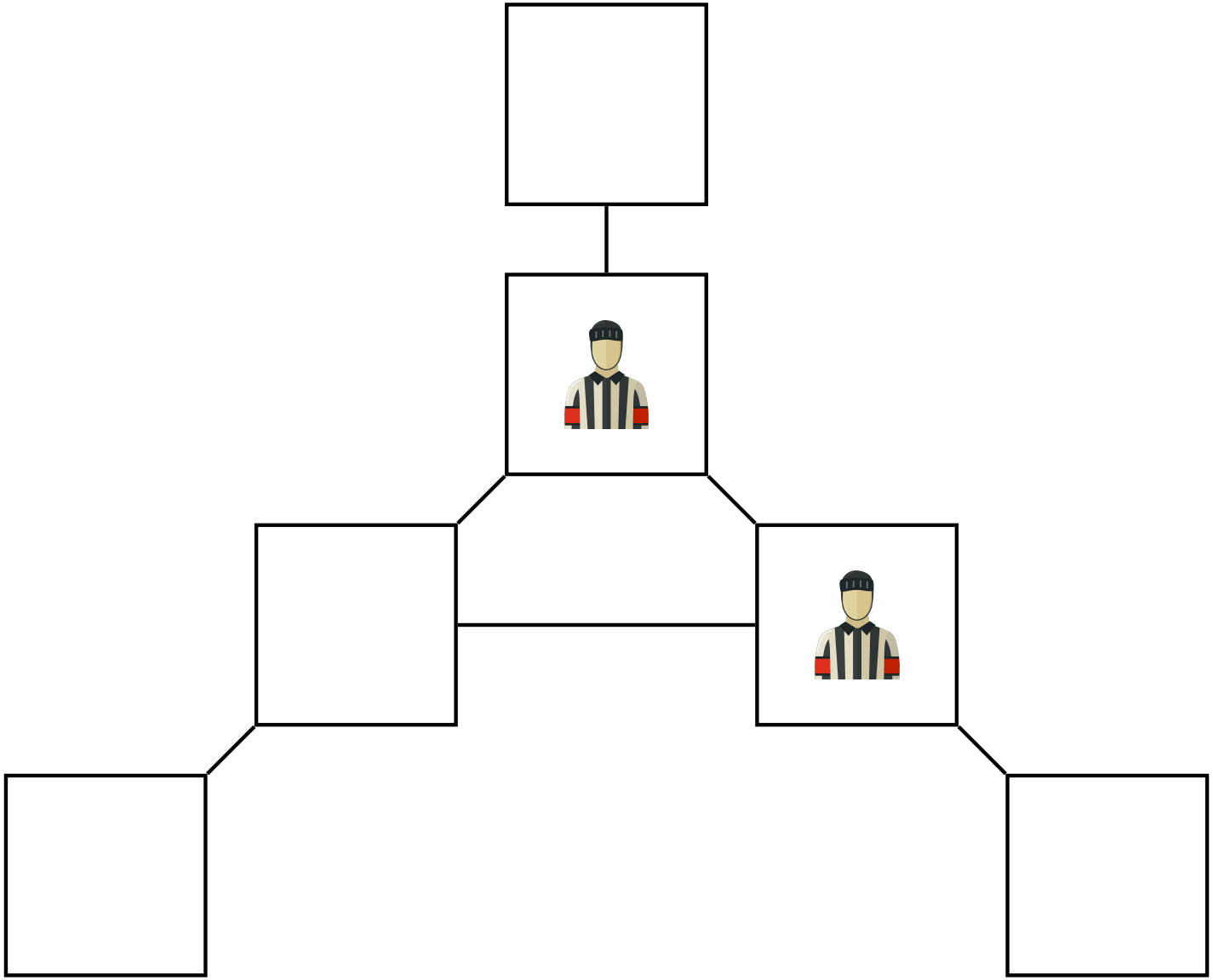

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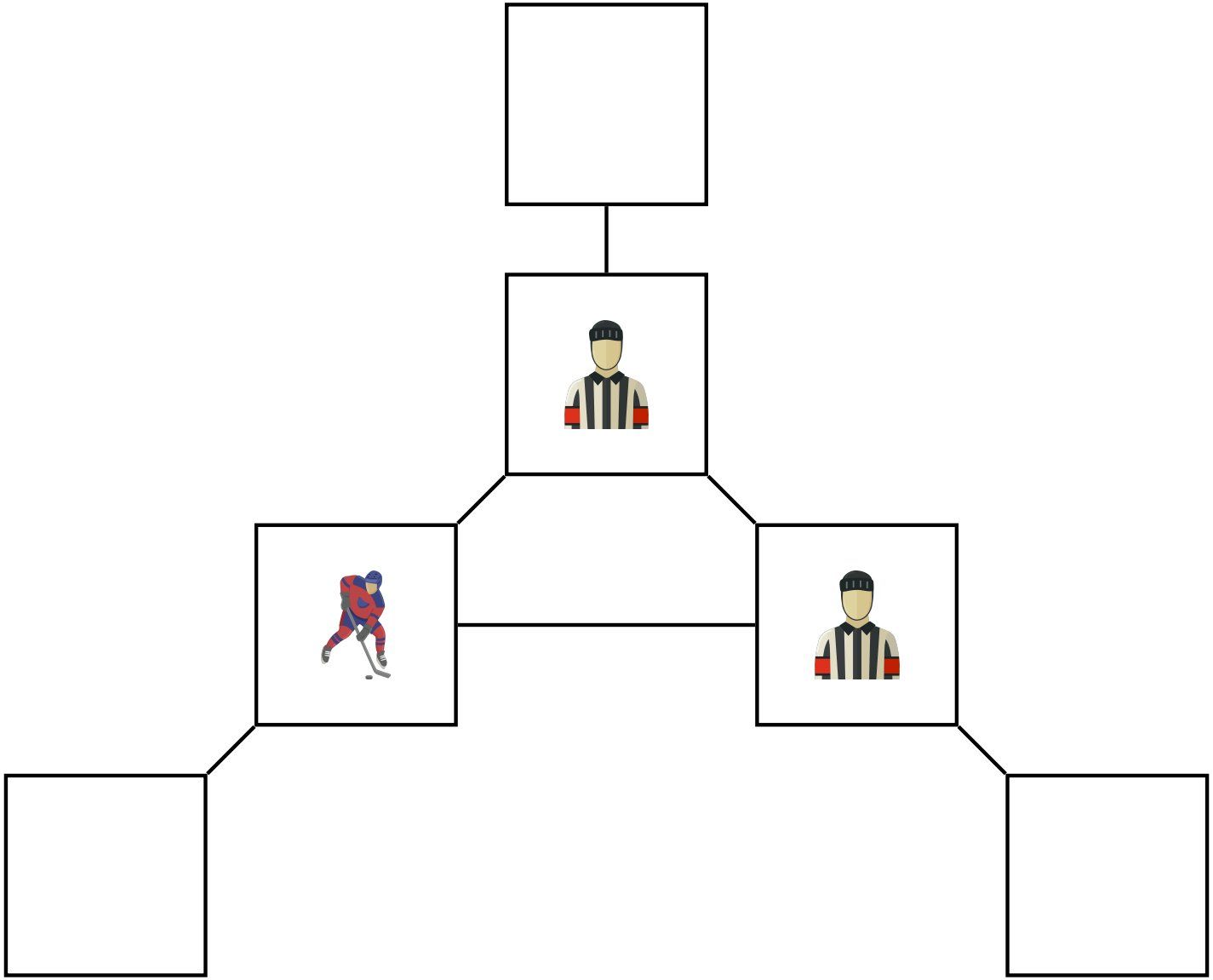

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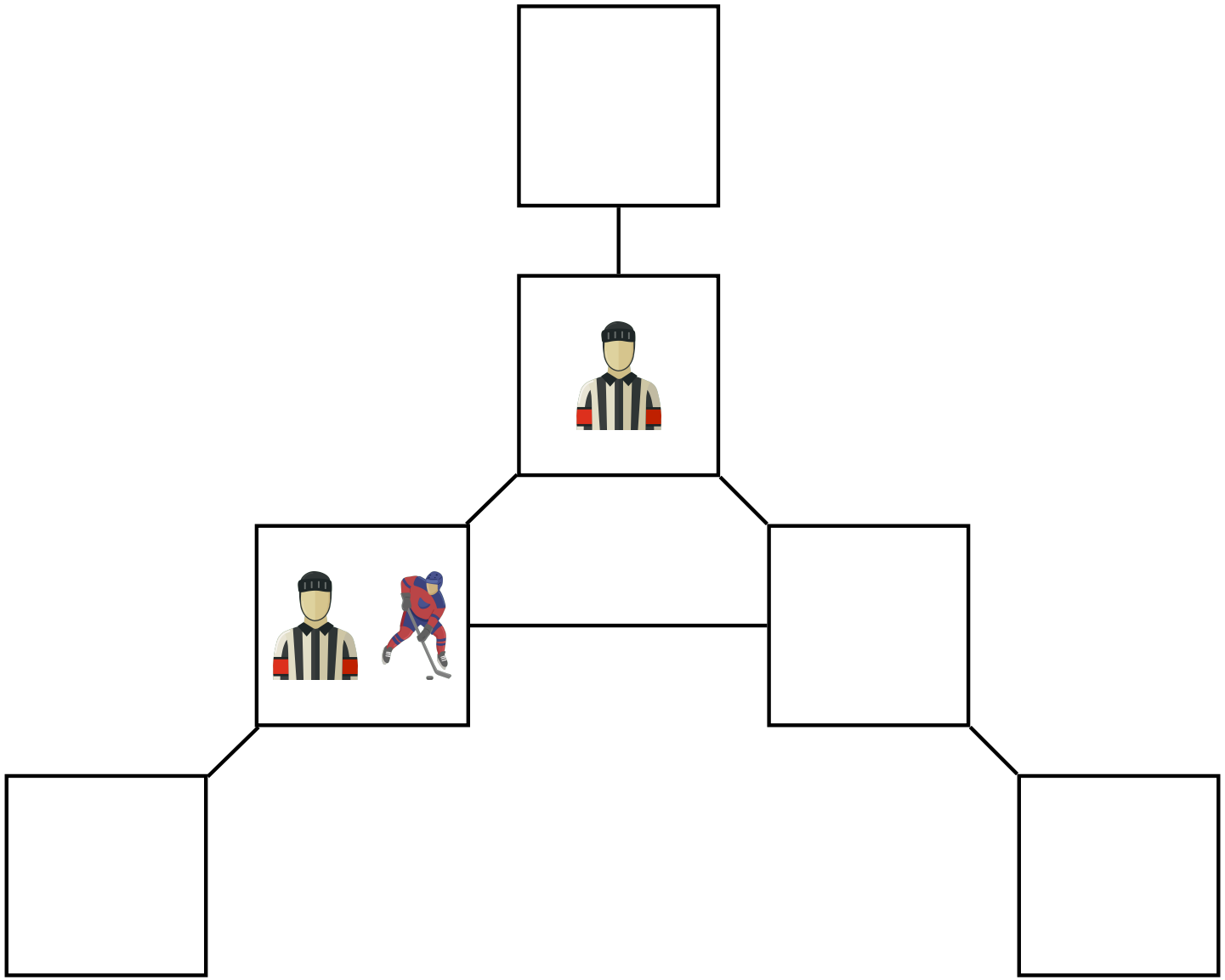

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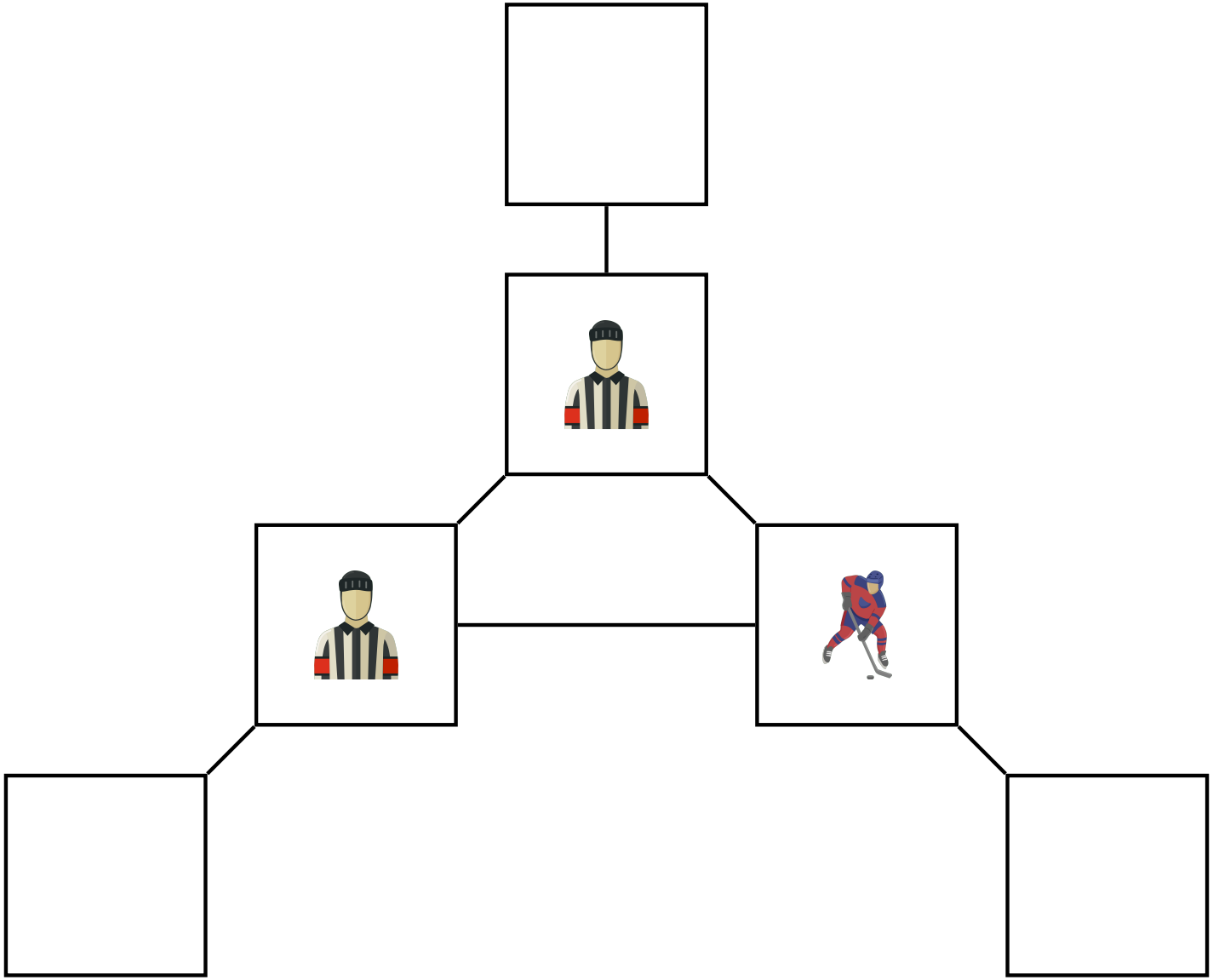

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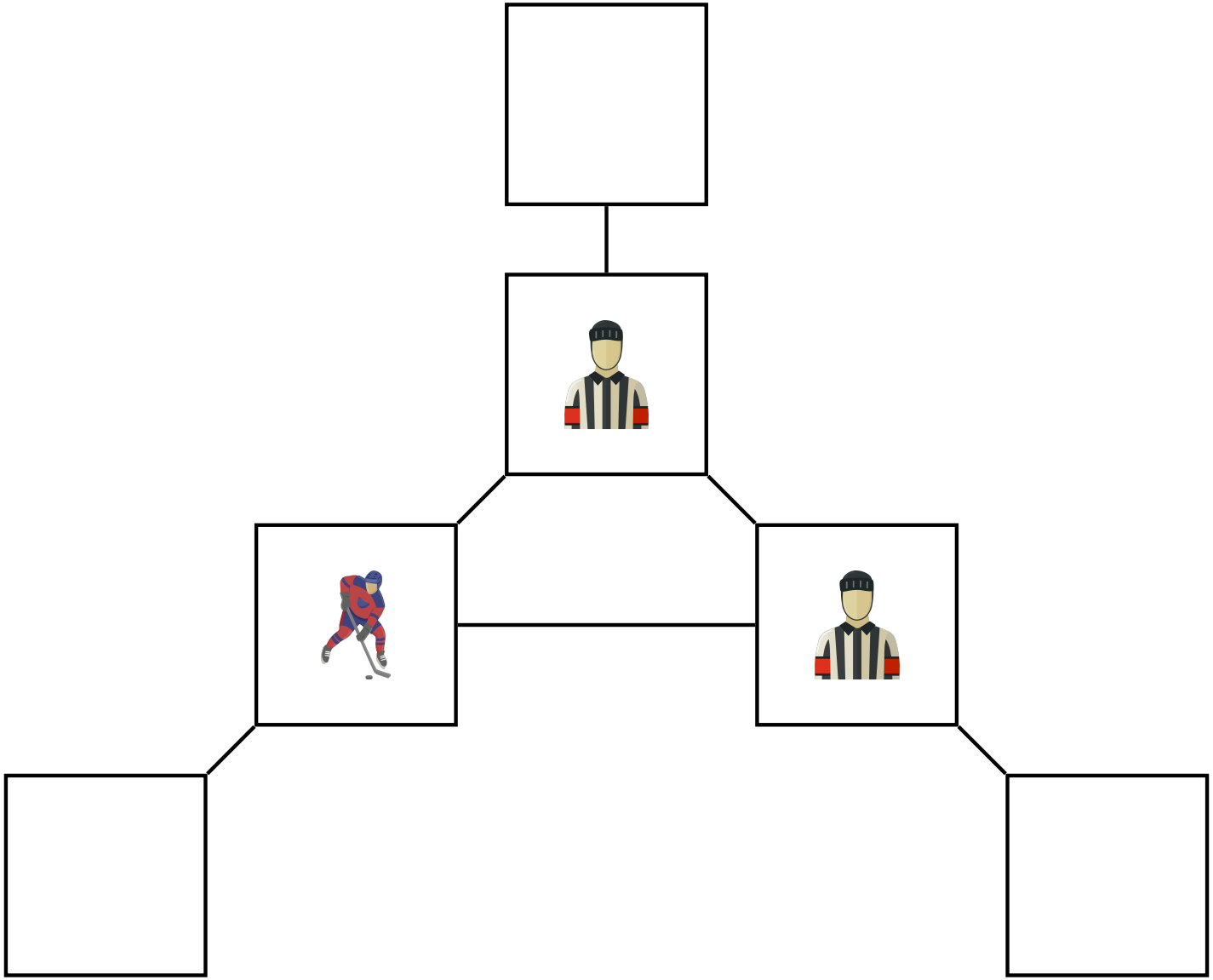

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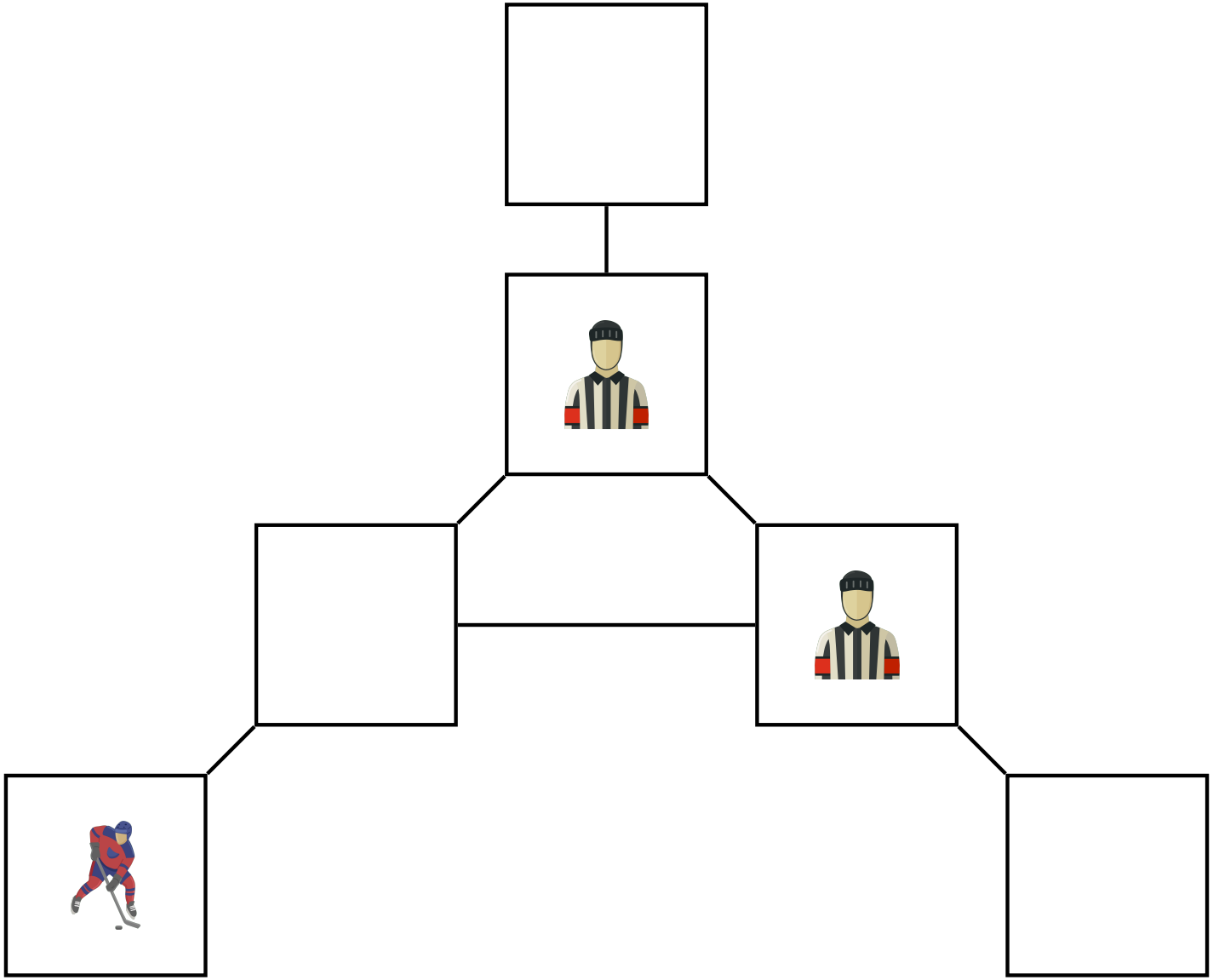

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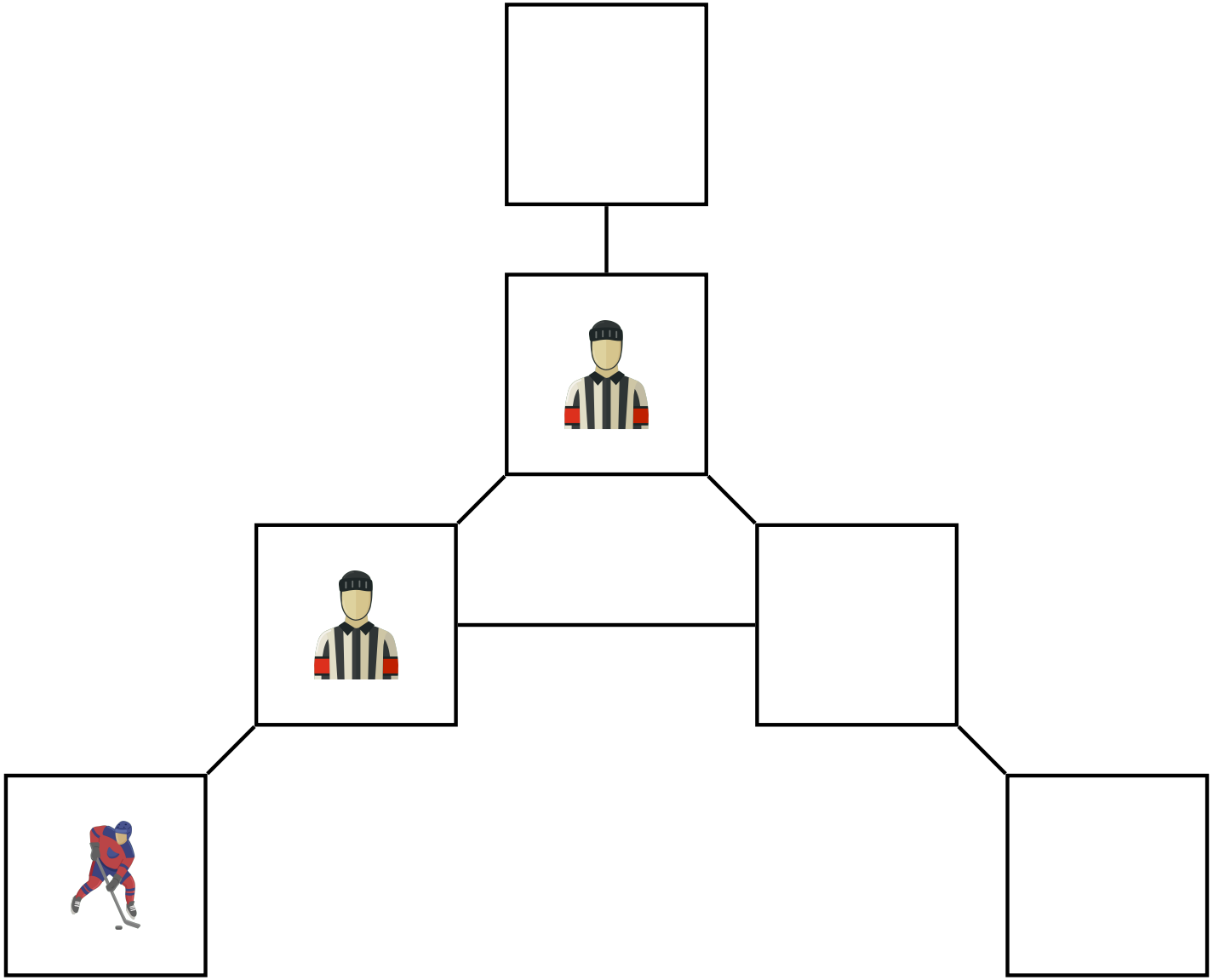

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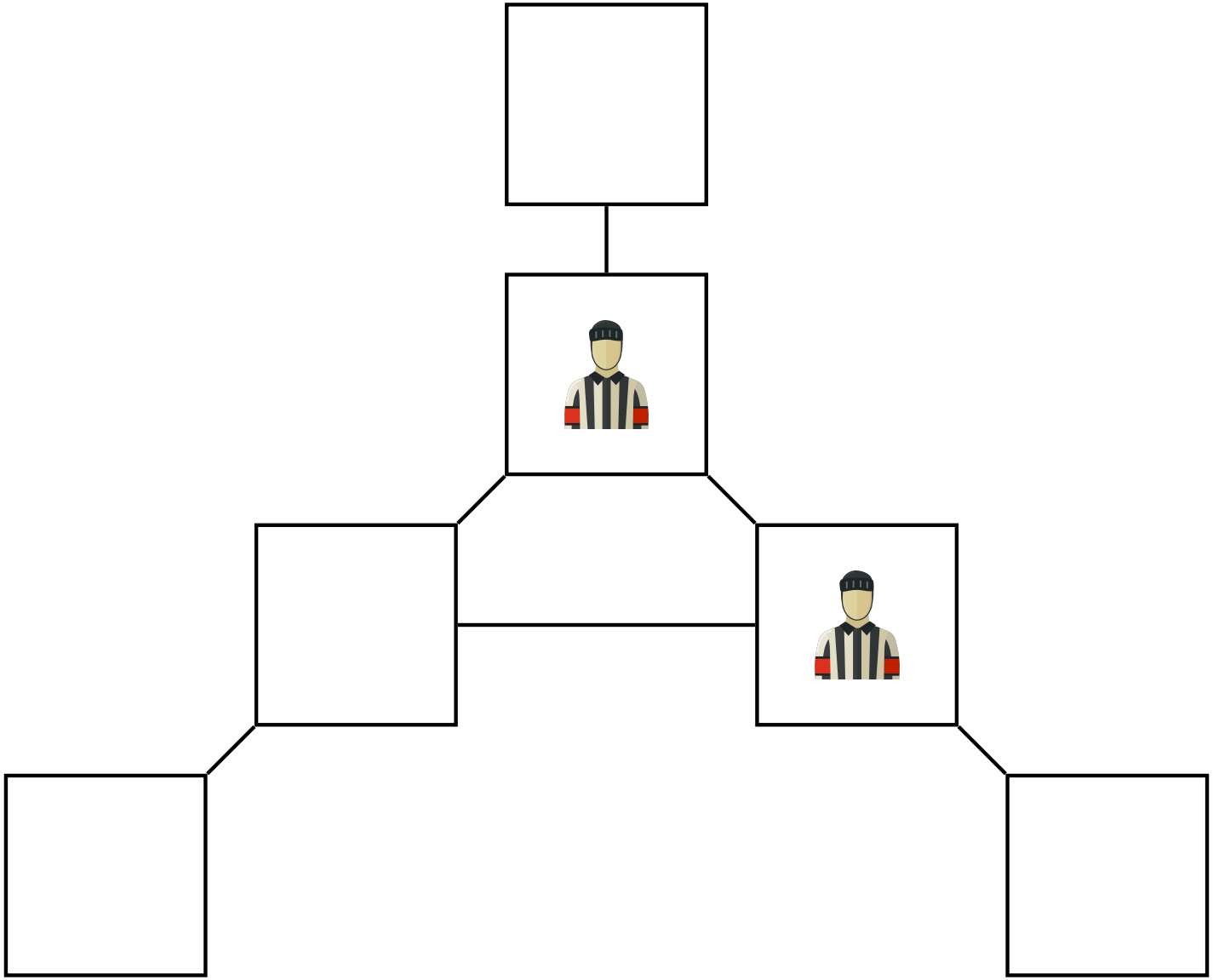
- Cops and Robber move at the same time, but the Robber knows what moves the Cops will make
- Robber loses when he shares a vertex with a Cop or moves along the same edge as a Cop in the same turn

Cheating Robot



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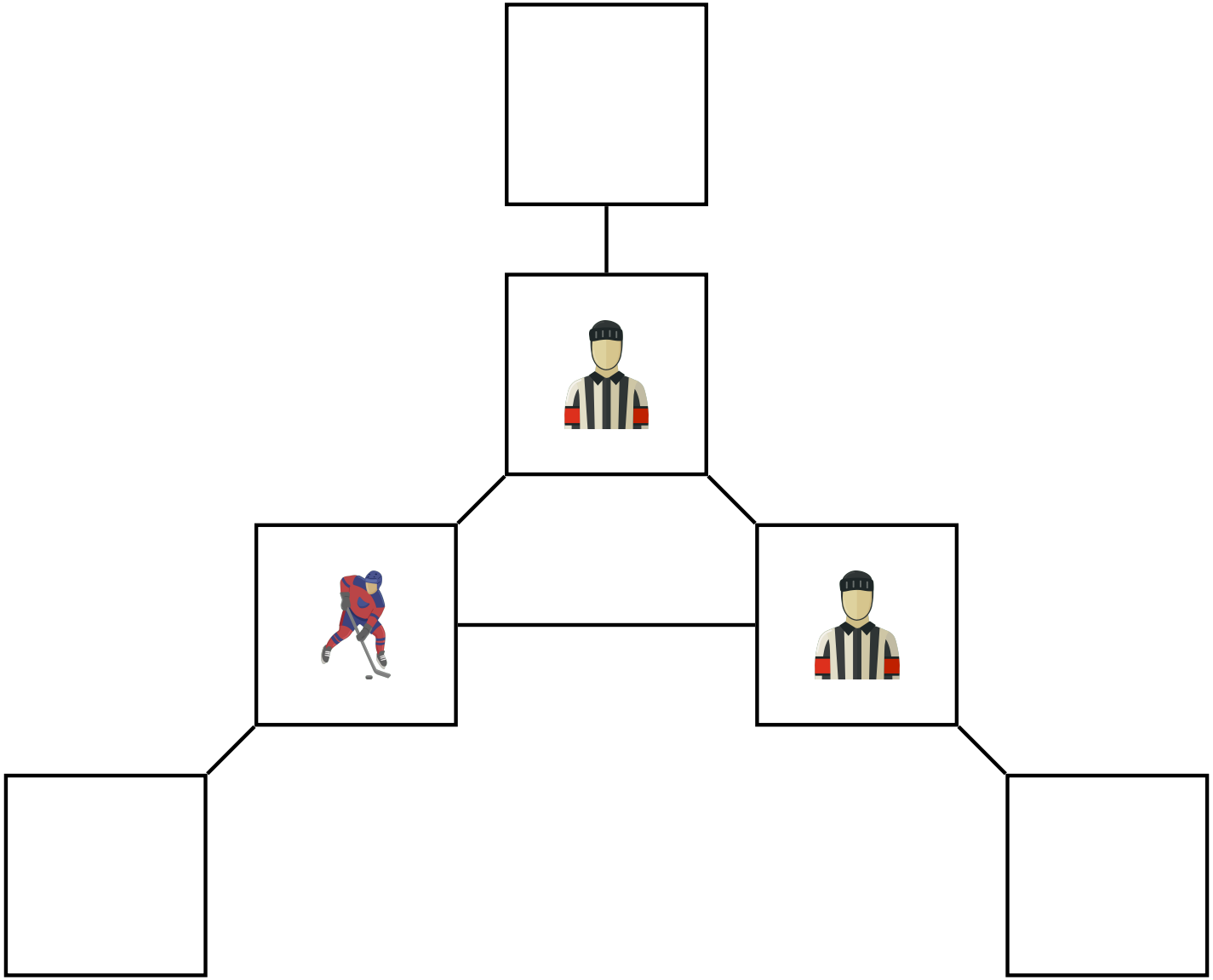

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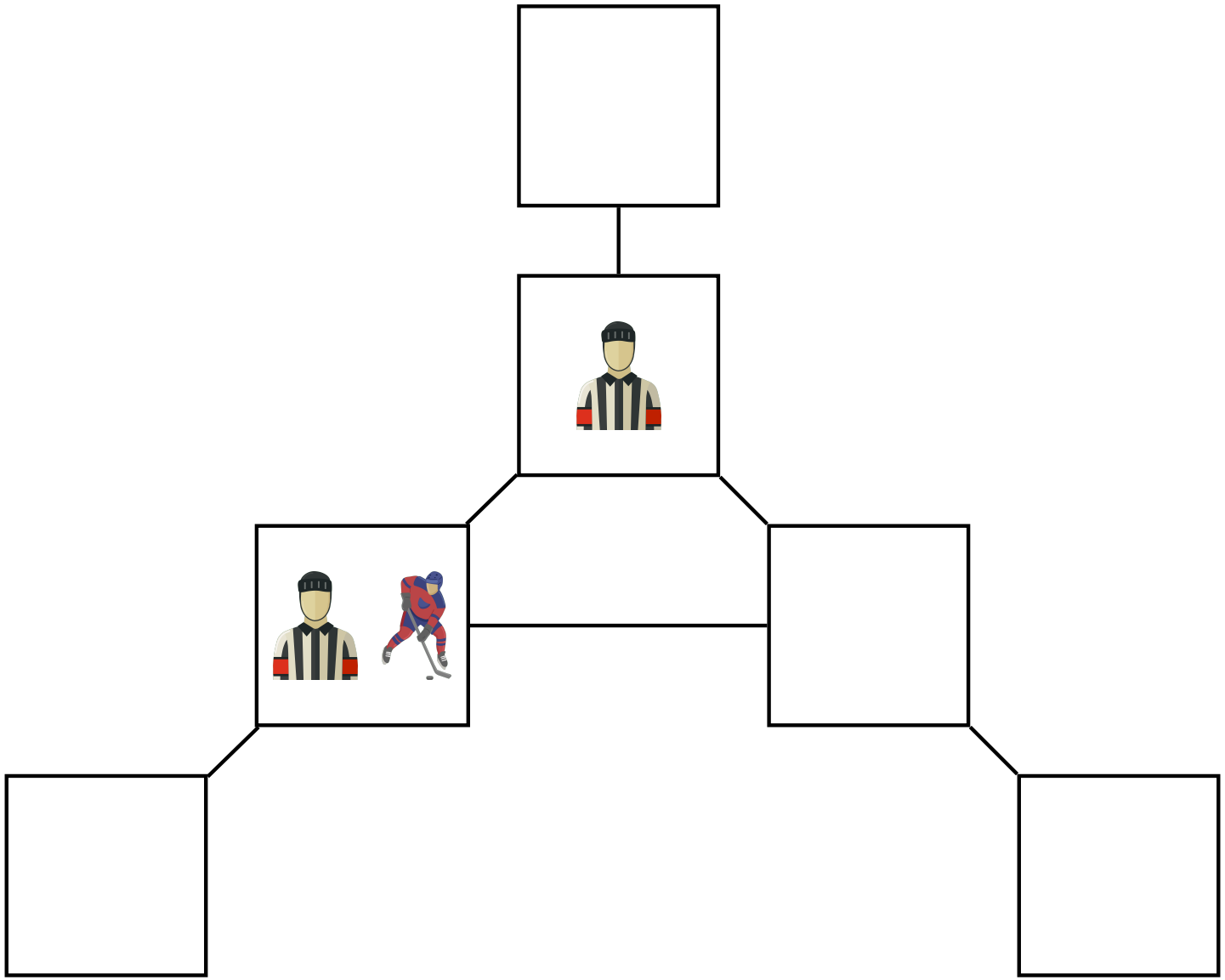

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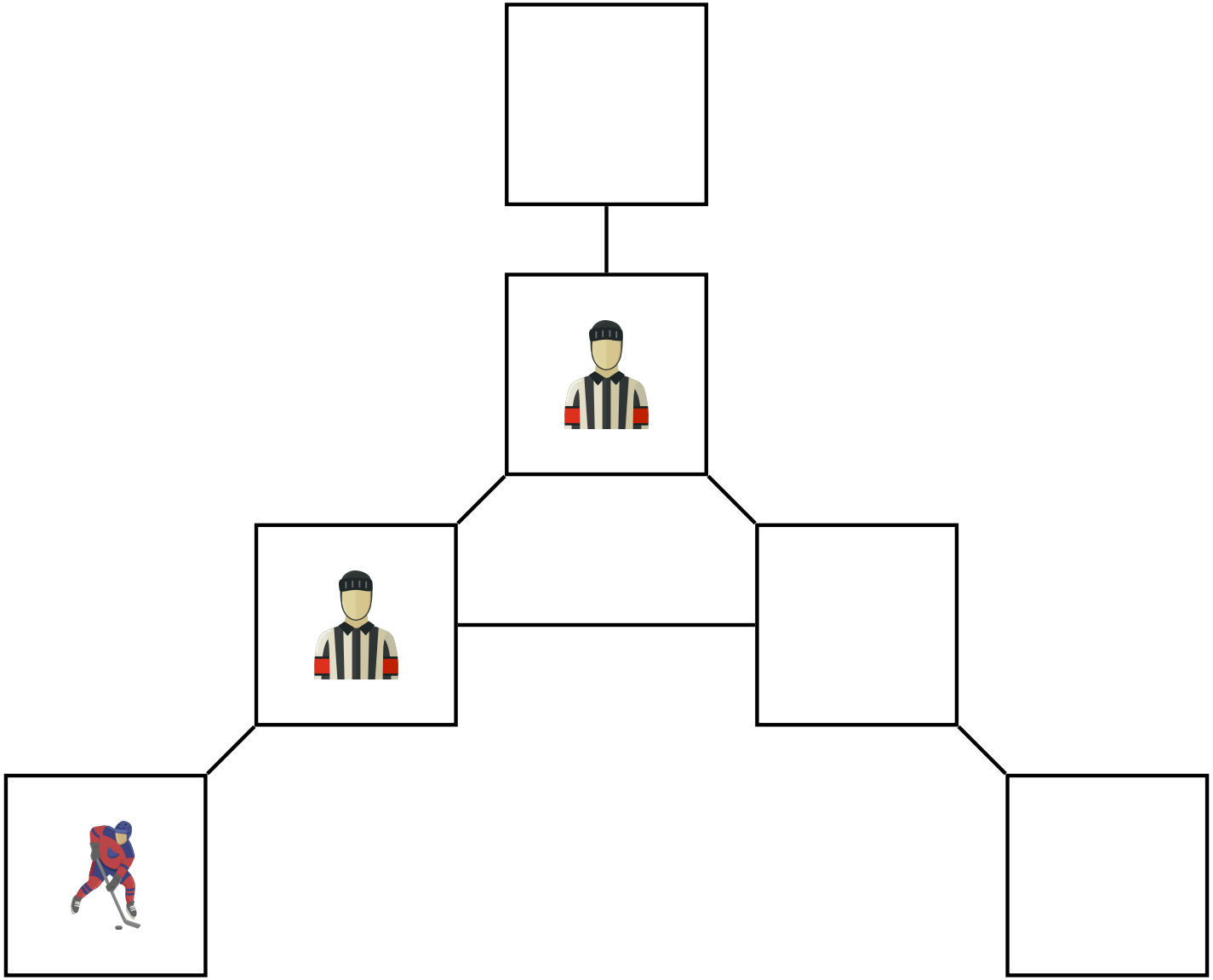

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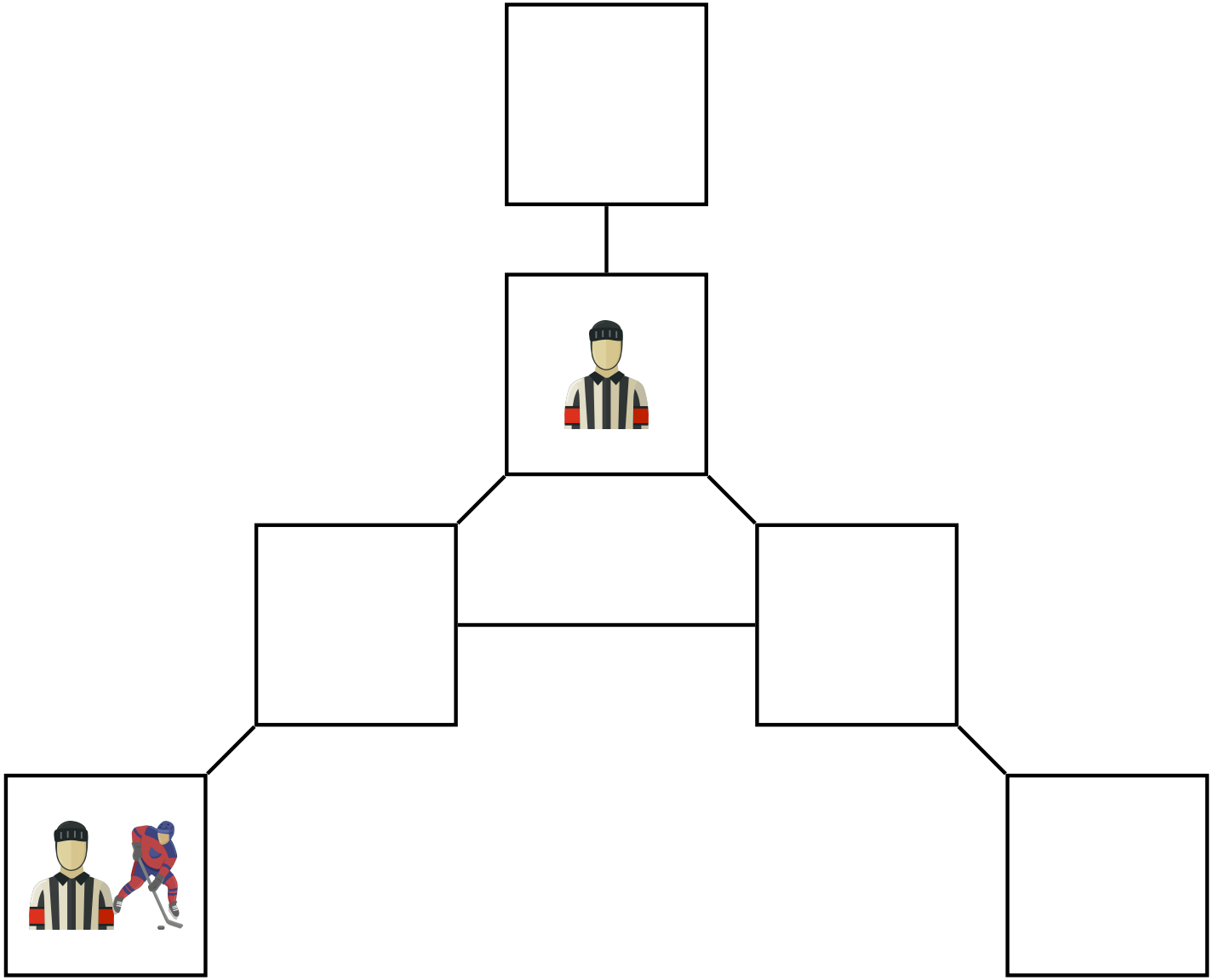

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- $2c_{cr}(G) \geq \sigma_a(G)$
- For all graphs G with minimum degree $\delta \geq 3$, if G has girth $g \geq 9$, then $\sigma_a(G) \geq 5$.

$\sigma_a(G) vs. c_{cr}(G)$

Is the active number always equal to the cheating number?

Building a Counterexample

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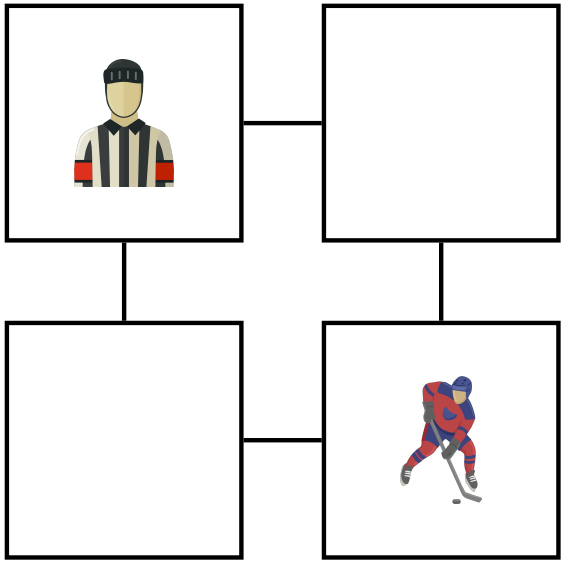
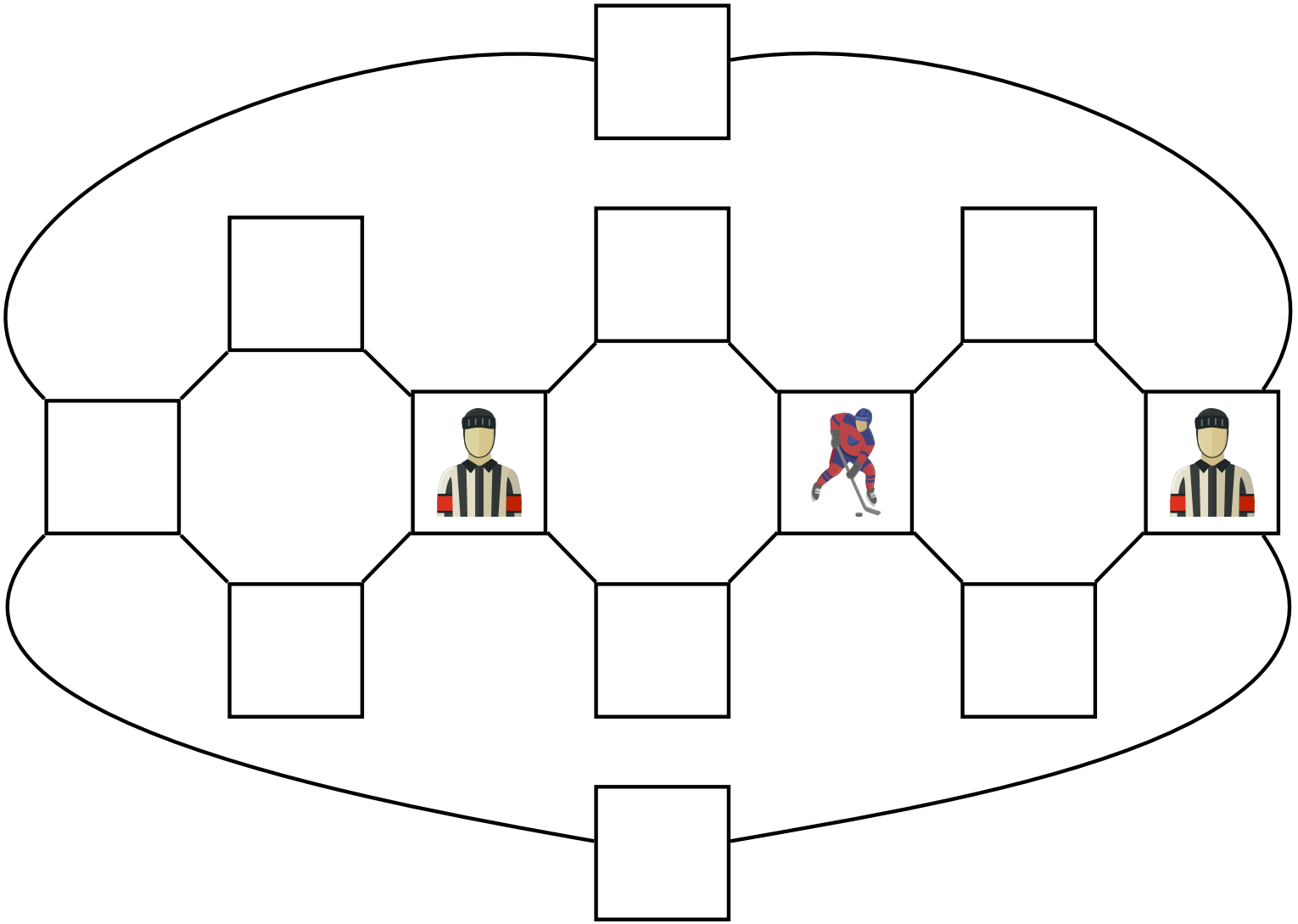


Active Surrounding



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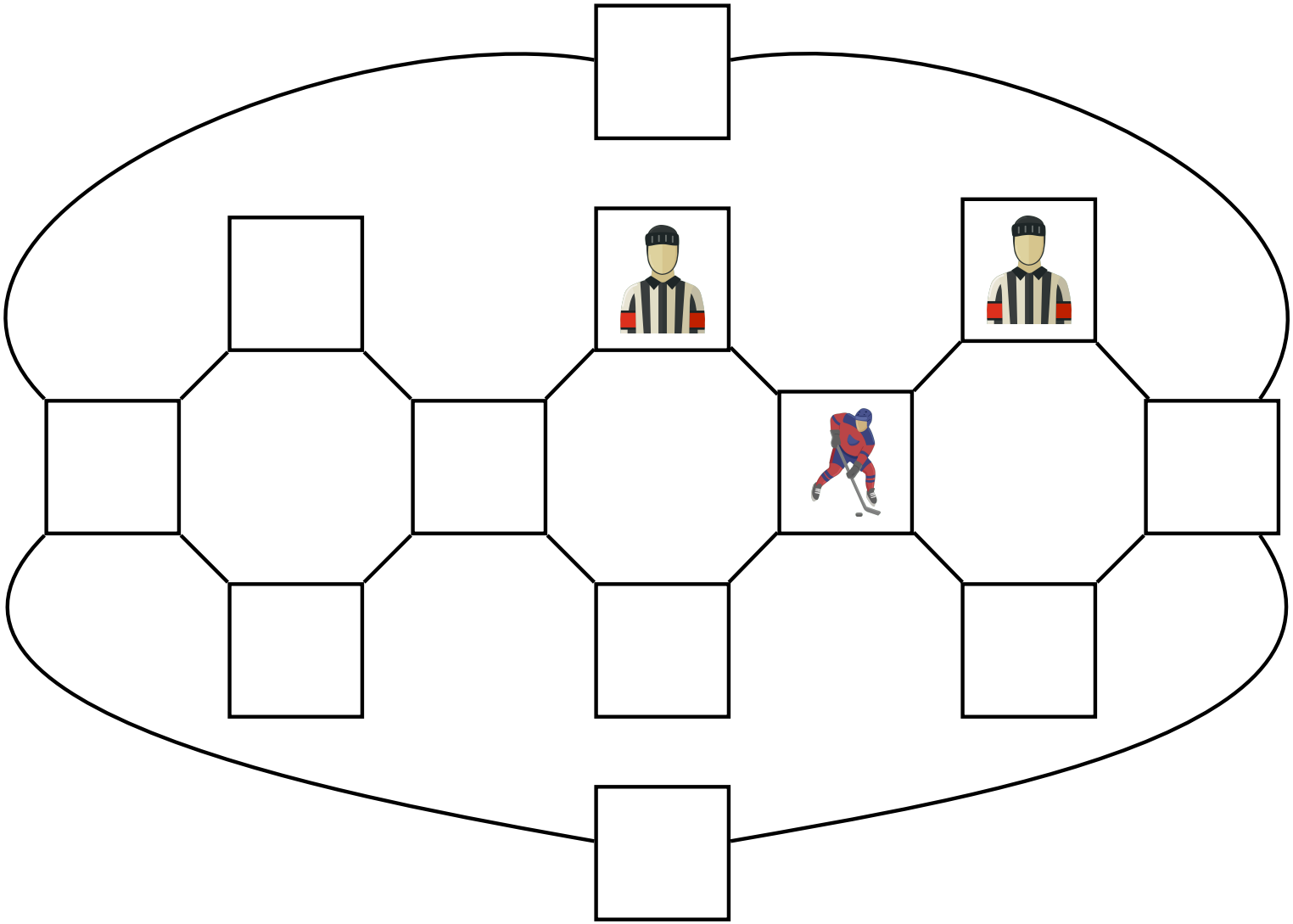

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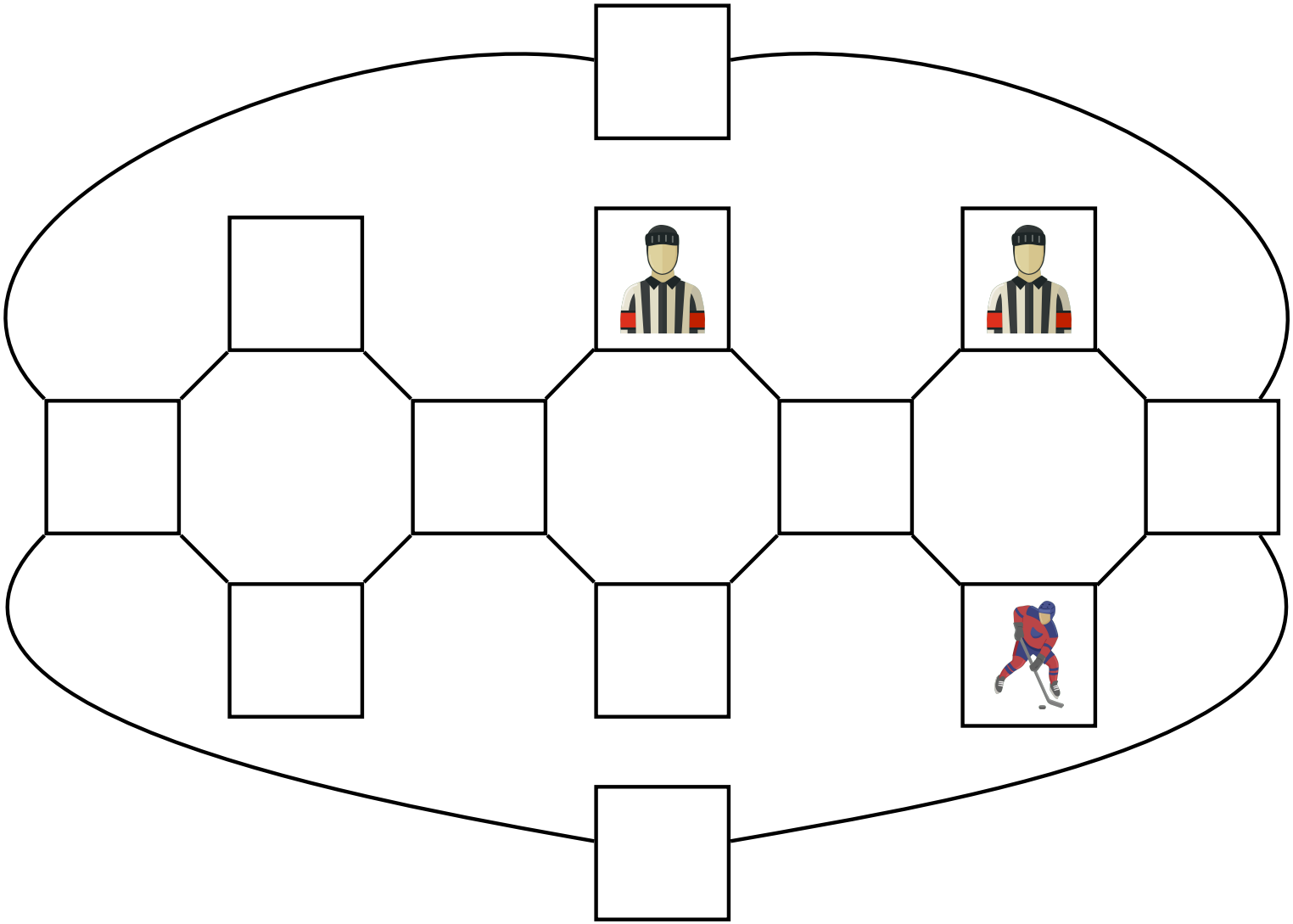

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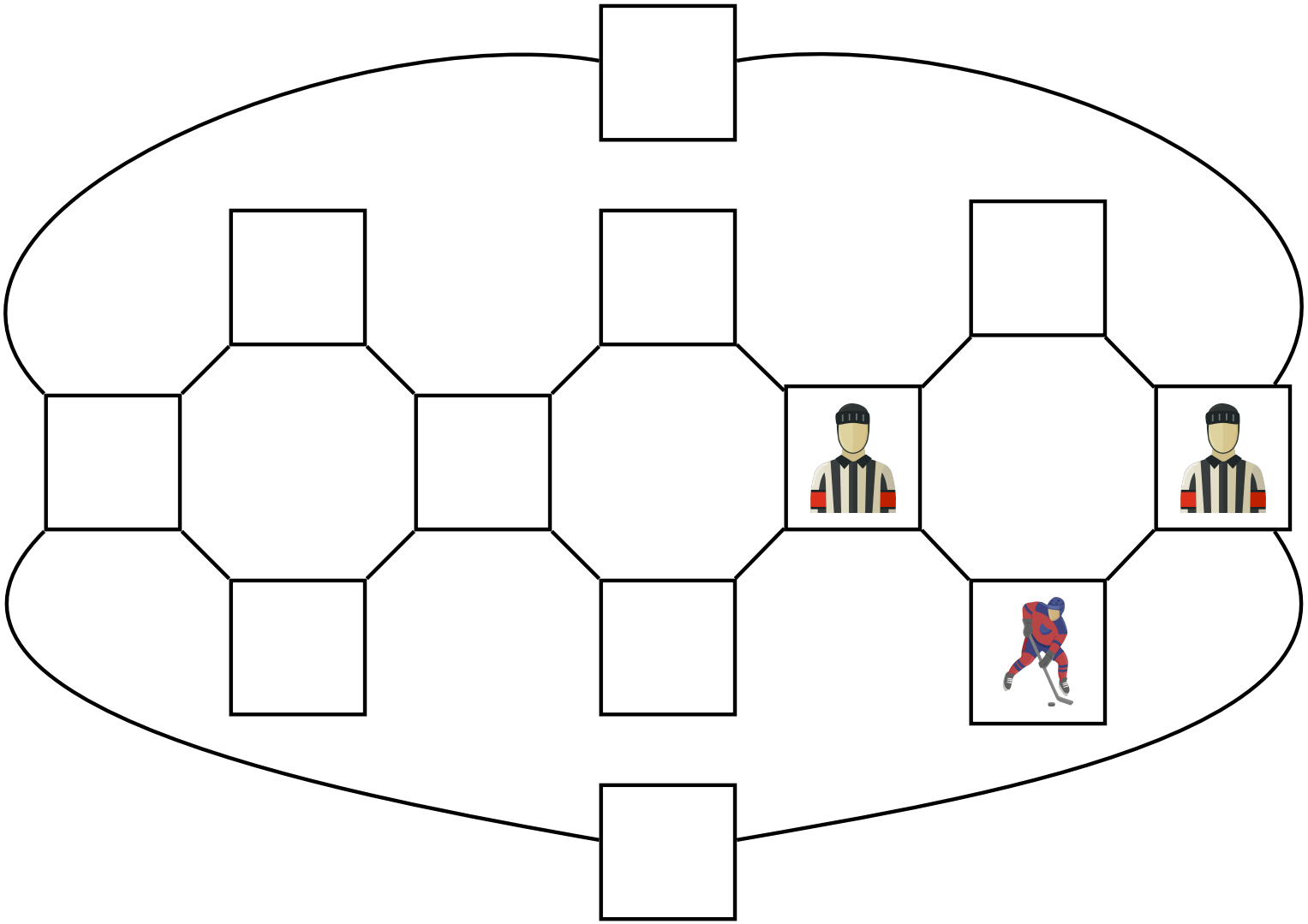

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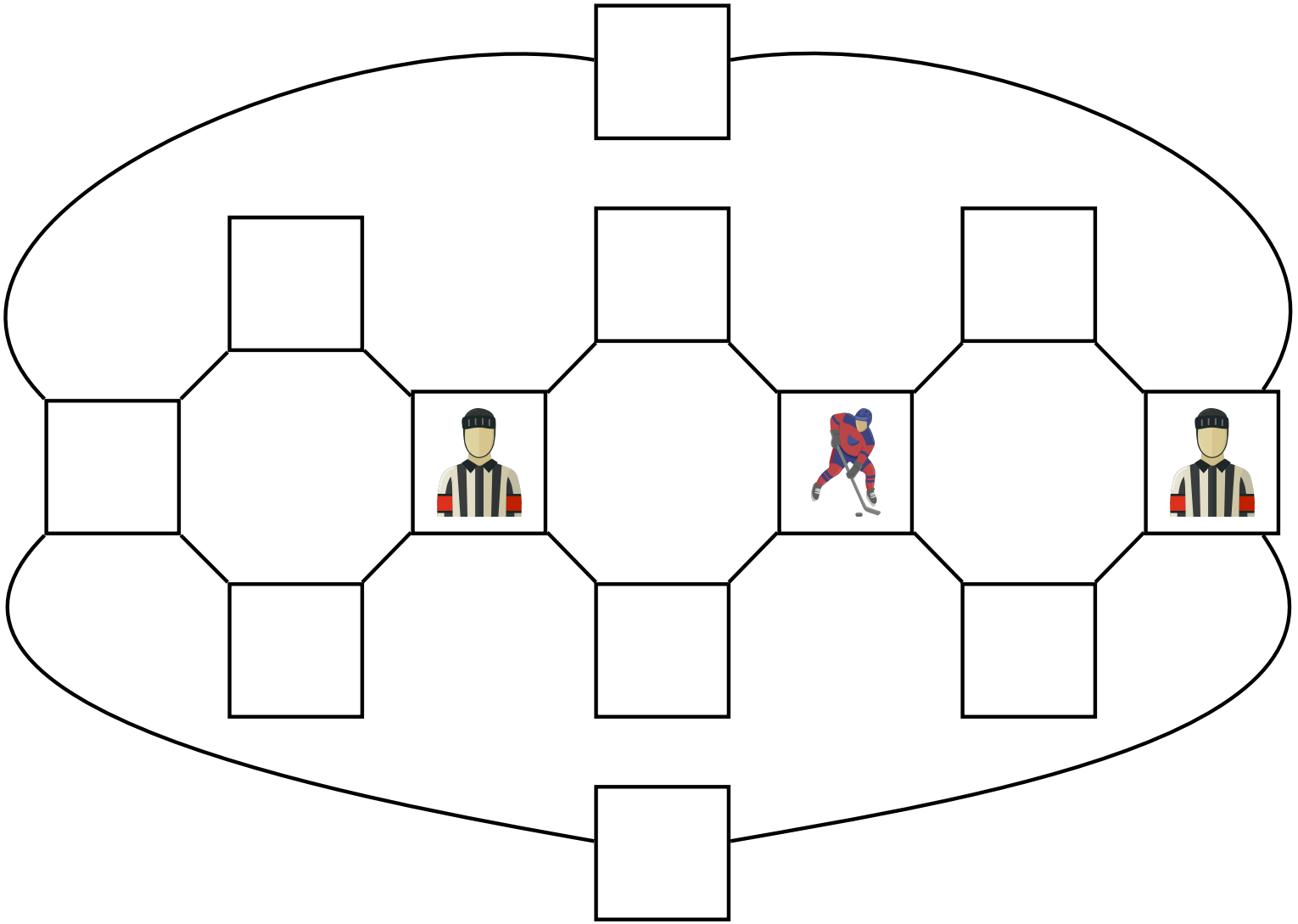

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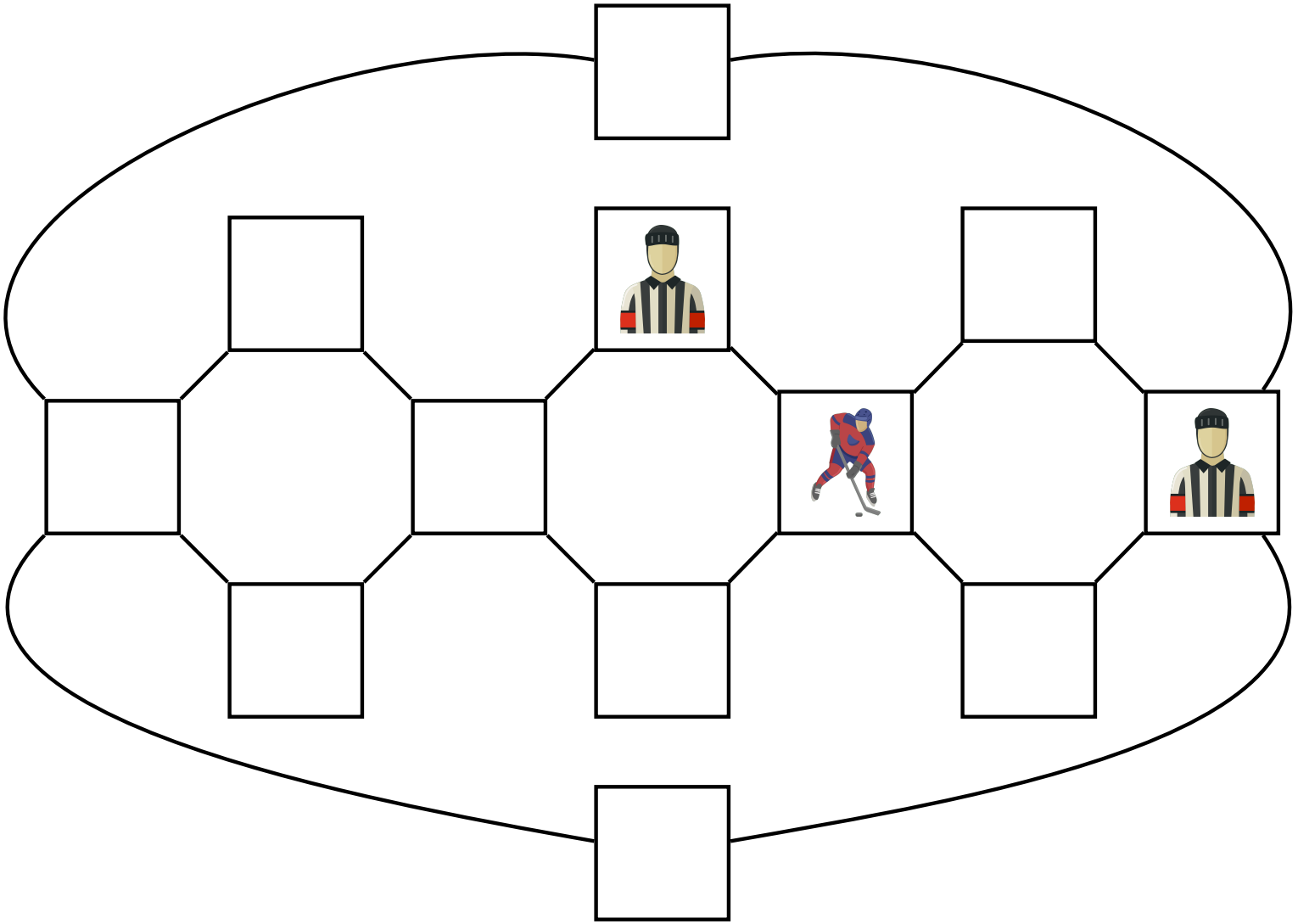

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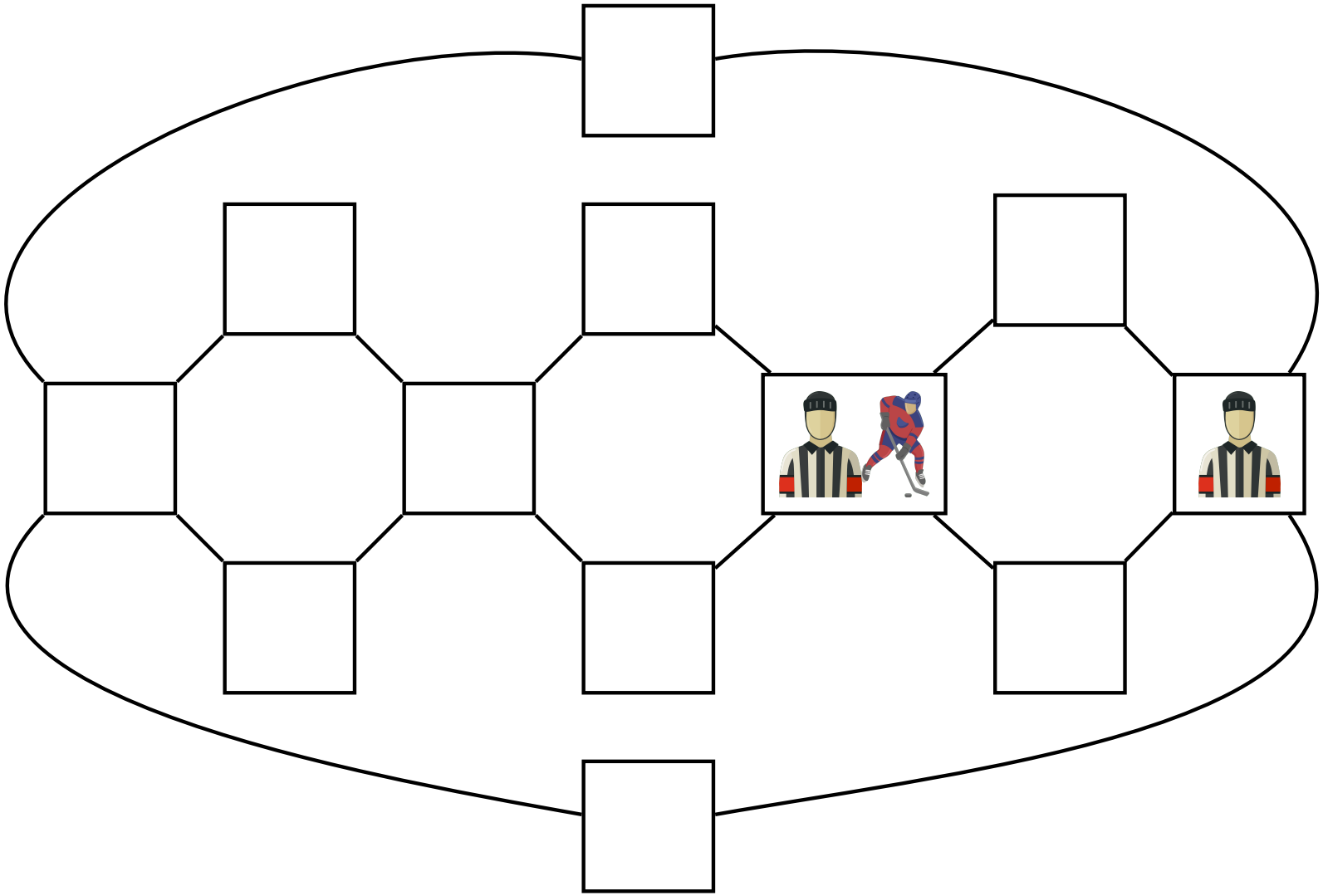

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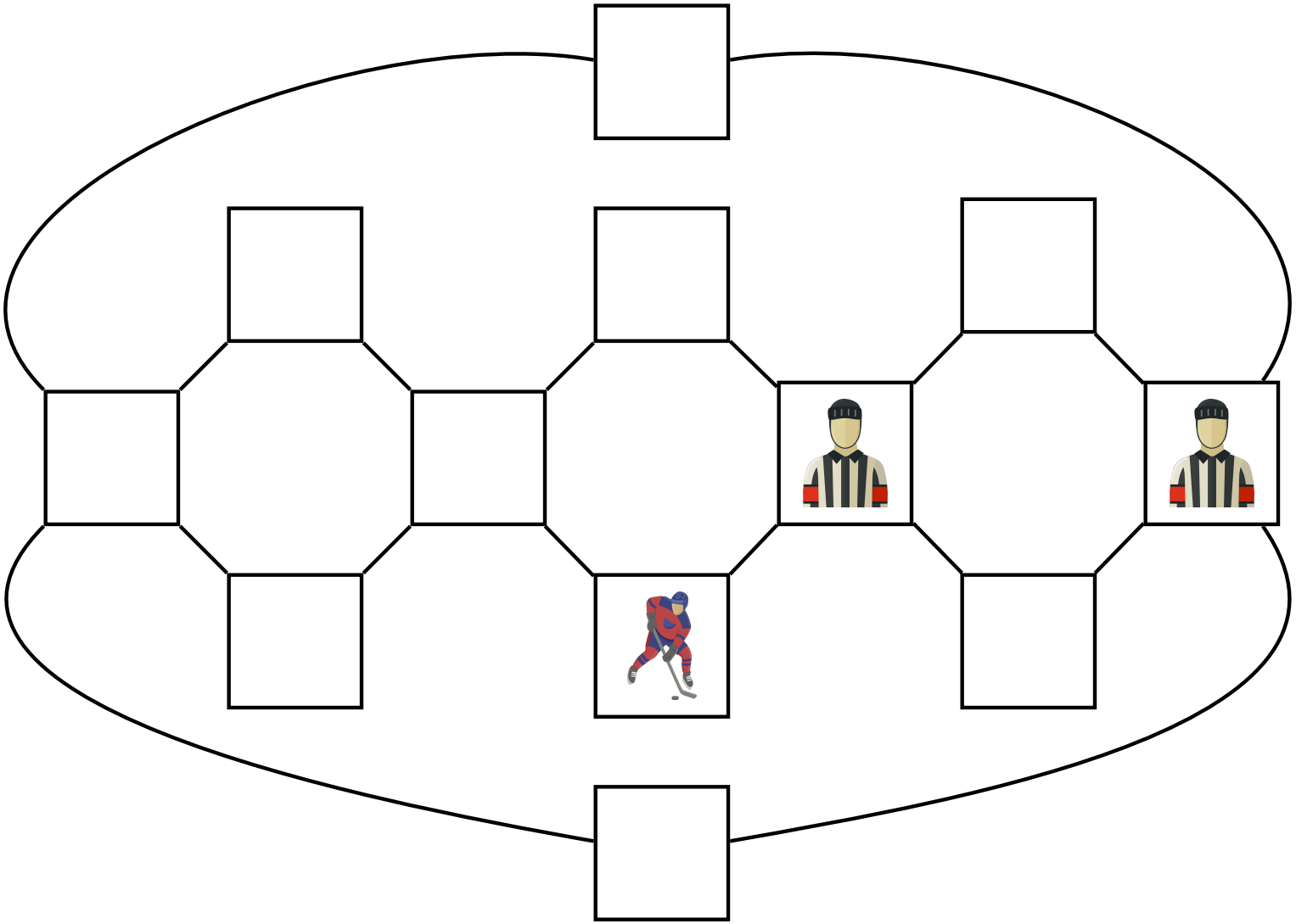

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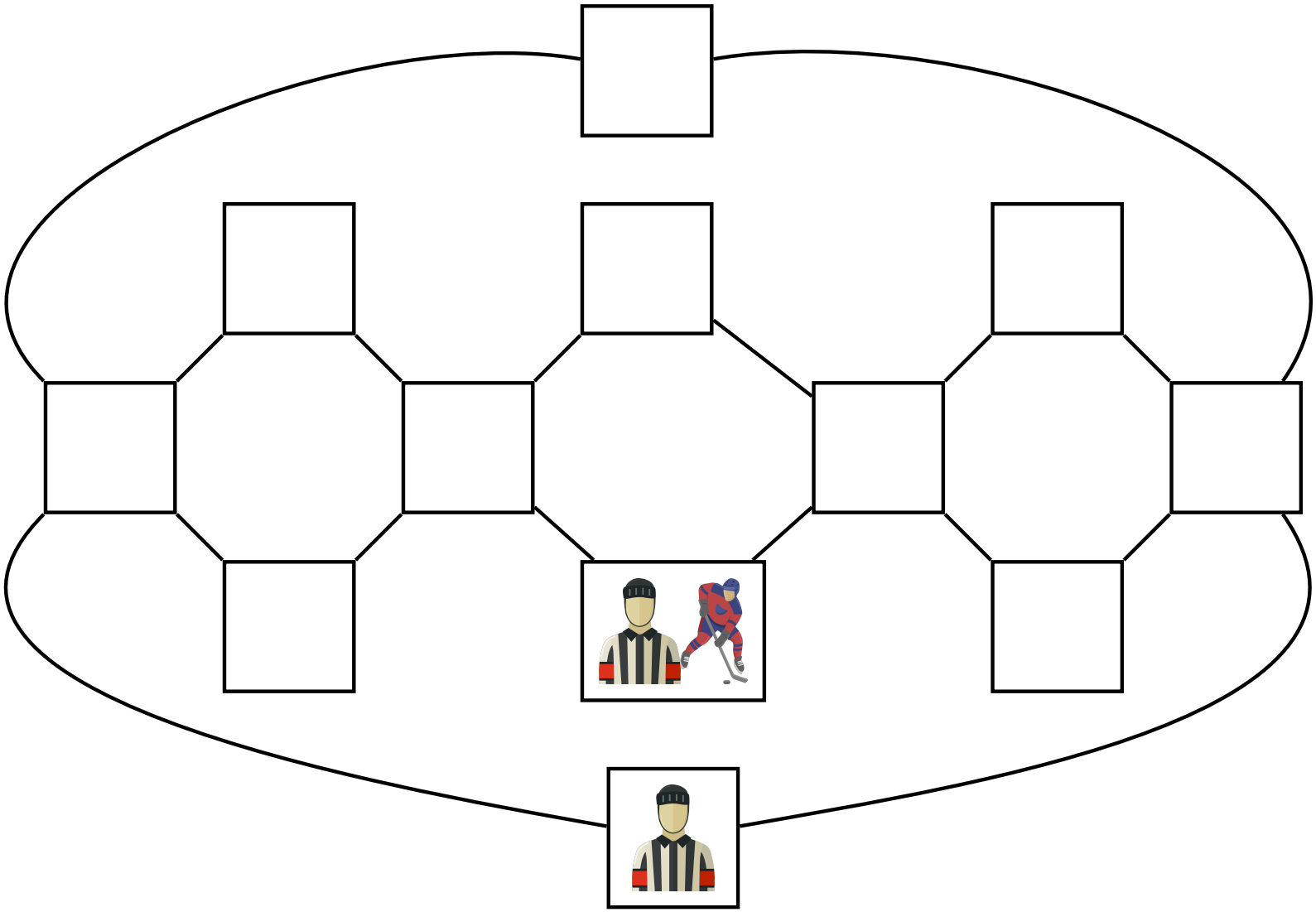

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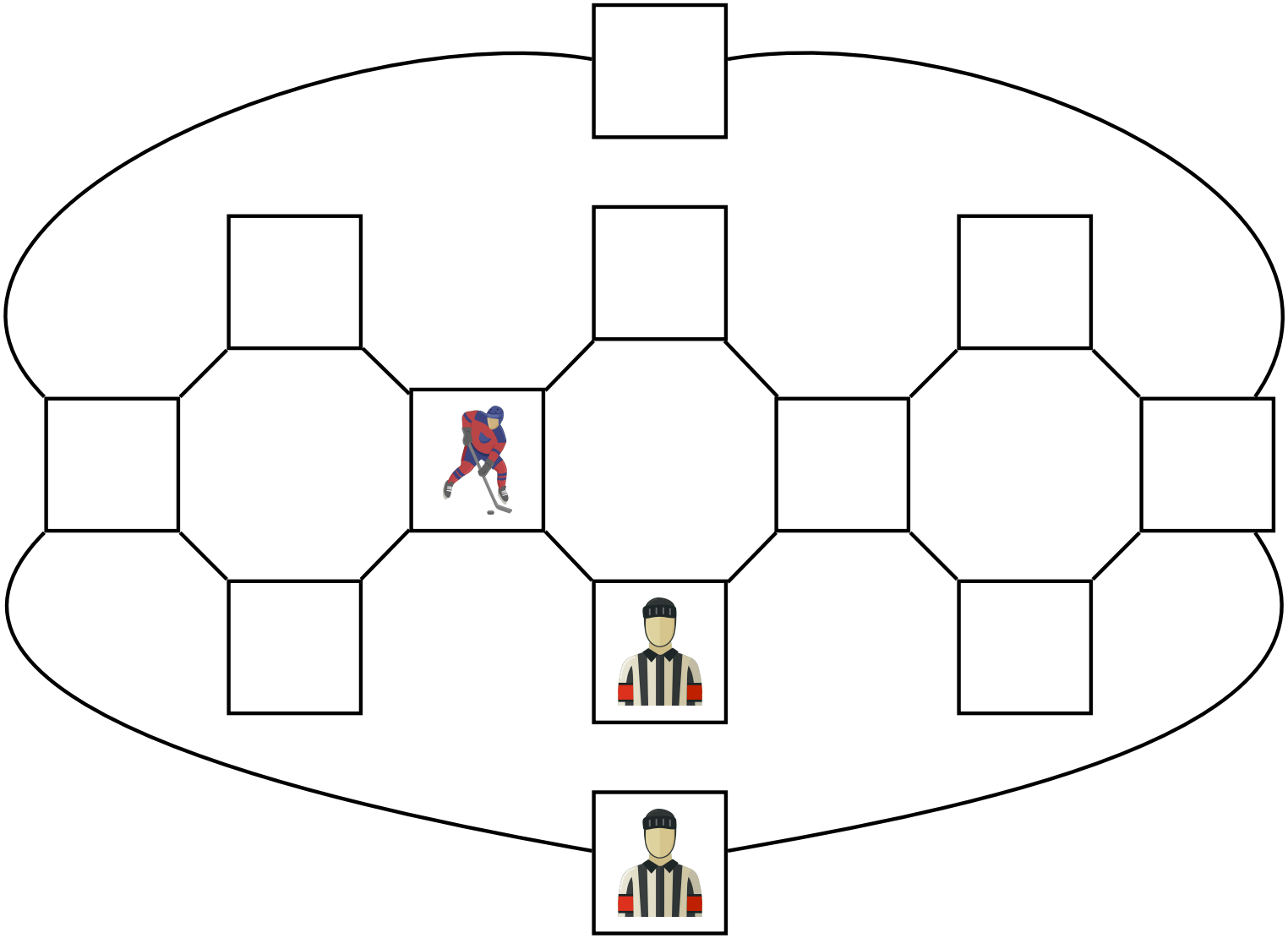
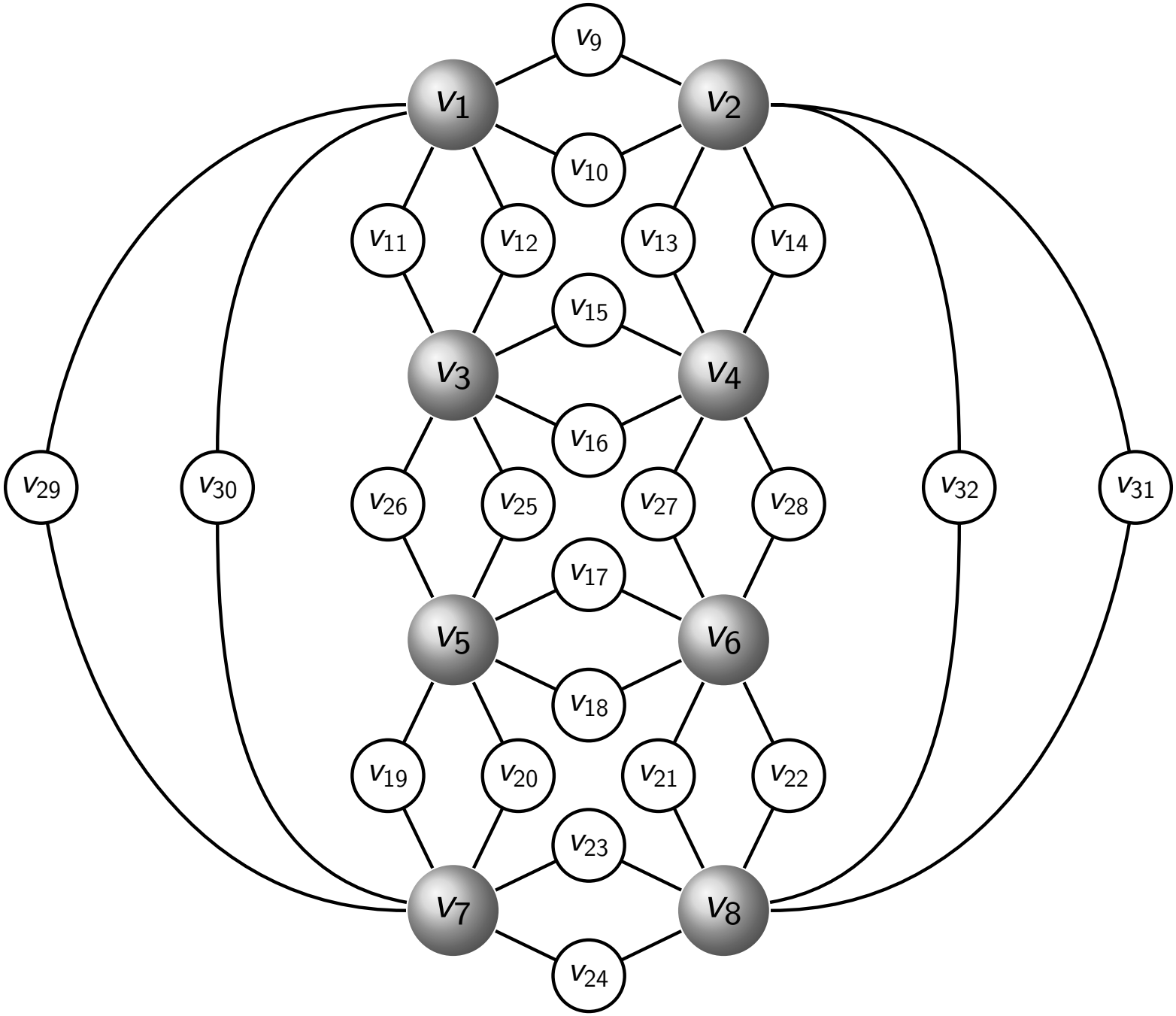


The subdivision graph $S(G)$ of a graph G is the graph formed by subdividing every edge in G once. We define the *double subdivision graph*, $DS(G)$, as the graph formed by the following two steps:

- Duplicate every edge in G (call this multi-graph G')
- Subdivide every edge in G'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