# 10th Graph Searching in Canada (GRASCan) Workshop


August 3–5, 2022
Online

# Wednesday, August 3
# Summer School

All times EDT

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|---|---|
| 9:30 - 12:00 | **Shahin Kamali**, York University<br>*An introduction to the graph burning problem* |
| 12:00-1:00 | LUNCH |
| 1:00 – 3:30 | **Anthony Bonato**, Toronto Metropolitan University<br>*An introduction to the cop number of a graph* |

# Abstracts of Summer School Sessions

**Shahin Kamali**, York University — Wednesday
*An introduction to the graph burning problem* — 9:30 - 12:00

Graph burning is a simple model for the spread of social influence in networks. The objective is to measure how quickly a "fire," for example, a piece of fake news, can be spread in a network. The graph burning problem has been the subject of considerable research in recent years. In this session, we review the main results on the graph burning problem. Topics include lower and upper bounds, the "graph burning conjecture", exact and approximation algorithms, complexity, and graph classes.

**Anthony Bonato**, Toronto Metropolitan University — Wednesday
*An introduction to the cop number of a graph* — 1:00 - 3:30

Cops and Robbers is one of the most well-studied pursuit-evasion games played on graphs. The main parameter in Cops and Robbers, the cop number, continues to be the focus of considerable research effort in graph theory. In this session, we will give an introduction to and overview of the main results on the cop number. Topics will range from bounds, algorithms, and variants.

# Thursday, August 4

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| 9:45 - 10:00 | Opening remarks |
| 10:00 - 11:00 | **Bojan Mohar**, Simon Fraser University<br>*The game of Cops and Robber on geodesic spaces* |
| 11:00 - 11:30 | BREAK |
| 11:30 - 12:00 | **Petra Wolf**, Universität Trier<br>*Cop and Robber Played on Edge Periodic Temporal Graphs* |
| 12:00 - 12:30 | **Todd Mullen**, Acadia University/St. Francis Xavier University<br>*Surrounding an Active Robber* |
| 12:30 - 2:00 | LUNCH |
| 2:00 - 2:30 | **Ryan Cushman**, Toronto Metropolitan University<br>*On Meyniel Extremal Families of Graphs* |
| 2:30 - 3:00 | **Ben Cameron**, The King's University<br>*The node cop-win reliability of a graph* |
| 3:00 - 4:00 | OPEN DISCUSSION |

# Friday, August 5

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| 10:00 - 11:00 | **Melissa Huggan**, Vancouver Island University<br>*Pursuit-Evasion Games: New Results and Open Problems* |
| 11:00 - 11:30 | COFFEE |
| 11:30 - 12:00 | **Trent Marbach**, Toronto Metropolitan University<br>*Pursuit-evasion games on latin square graphs* |
| 12:00 - 12:30 | **John Marcoux**, Memorial University of Newfoundland<br>*Firefighting with a Distance-Based Restriction* |
| 12:30 - 2:00 | LUNCH |
| 2:00 - 2:30 | **Caleb Jones**, Memorial University of Newfoundland<br>*Extending Graph Burning to Hypergraphs* |
| 2:30 - 3:00 | **Mohamed Omar**, Harvey Mudd College<br>*Burning Graph Classes* |
| 3:00 - 4:00 | OPEN DISCUSSION |

# Abstracts of Talks

**Bojan Mohar**, Simon Fraser University — Thursday
*The game of Cops and Robber on geodesic spaces* — 10:00 - 11:00

The game of Cops and Robber is traditionally played on a finite graph. The purpose of this paper is to introduce and analyse the game that is played on an arbitrary geodesic space (a compact, path-connected space endowed with intrinsic metric). It is shown that the game played on metric graphs is essentially the same as the discrete game played on abstract graphs and that for every compact geodesic surface there is an integer $c$ such that $c$ cops can win the game against one robber, and $c$ only depends on the genus $g$ of the surface. It is shown that $c = 3$ for orientable surfaces of genus 0 or 1 and nonorientable surfaces of crosscap number 1 or 2 (with any number of boundary components) and that $c = O(g)$ and that $c = \Omega(\sqrt{g})$ when the genus $g$ is larger. Our main motivation for discussing this game is to view the cop number (the minimum number of cops needed to catch the robber) as a new geometric invariant describing how complex is the geodesic space.

**Petra Wolf**, Universität Trier — Thursday
*Cop and Robber Played on Edge Periodic Temporal Graphs* — 11:30 - 12:00

We consider the cops and robber game variant consisting of one cop and one robber on edge periodic temporal graphs. In edge periodic temporal graphs, for each edge $e$, a binary string $s_e$ determines in which time step the edge is present, namely the edge $e$ is present in time step $t$ if and only if the string $s_e$ contains a 1 at position $t \bmod |s_e|$. This periodicity allows for a compact representation of the sequence of snapshot graphs. We proof that even for very simple underlying graphs, i.e., directed and undirected cycles the problem whether a cop-winning strategy exists is NP-hard and W1-hard parameterized by the number of vertices. We give lower bounds, matching the upper bounds from Erlebach and Spooner in 2020, for the ratio between the length of the underlying cycle and the least common multiple (LCM) of the lengths of binary strings describing edge-periodicies over which the graph is robber-winning. Further, we improve the previously known EXPTIME upper bound for Periodic Cop&Robber on general edge periodic temporal graphs to PSPACE-membership.

This talk is based on a joined work together with Nils Morawietz.

**Todd Mullen**, Acadia University/St. Francis Xavier University — Thursday
*Surrounding an Active Robber* — 12:00 - 12:30

The Surrounding Cop Number is a recently introduced parameter which measures the number of Cops required not to capture the Robber, but rather to occupy all vertices adjacent to the Robber (whether or not a Cop is on the Robber's vertex). In the Cops and Robber variant, Surrounding Cops and Robber, if a Cop ever lands on the Robber's vertex, the Robber doesn't lose, but rather he is simply compelled to move on his next turn to a vertex that doesn't contain a Cop. In this talk, we introduce two similar variants to Surrounding Cops and Robber, Active Robber and Fleeing Robber, that attempt to address certain peculiarities in the Cops' and Robber's respective strategies in Surrounding Cops and Robber. We conclude by discussing an interesting family of graphs on which the Active Number and Fleeing Number differ, and speculate the size of the largest possible difference between these two parameters on a given graph.

**Ryan Cushman**, Toronto Metropolitan University — Thursday 2:00 - 2:30
*On Meyniel Extremal Families of Graphs*

Meyniel's Conjecture states that there is a constant such that for any connected graph, the cop number is bounded above by this constant times the square root of the order. Although there are sublinear bounds on the cop number and verification for certain classes of graphs, in general there is a large gap between known bounds and Meyniel's Conjecture. Therefore, graph families that obtain this conjectured asymptotic upper bound, called Meyniel extremal families, can be of interest. In this talk, we provide new Meyniel extremal families and explore their properties. We provide new constructions with control over degree, chromatic number, and diameter. Employing methods from hypergraphs, we explore the degrees in families that are not Meyniel extremal and give the best-known upper bound on the cop number of vertex-transitive graphs with a prescribed degree. This is joint work with Anthony Bonato and Trent Marbach.

**Ben Cameron**, The King's University — Thursday 2:30 - 3:00
*The node cop-win reliability of a graph*

In this talk we present a new graph invariant called the node cop-win reliability. It is defined as the probability that the graph is cop-win if the vertices are operational independently at random with some fixed probability. This simple model is analogous to the well-studied notion of node reliability of a graph (which is the probability that the graph is connected under the same conditions). The node cop-win reliability also provides a measure of "cop-win-ness" of graphs under the variation of Cops and Robber where there are random intersection closures. We will provide the graphs that uniquely maximize the node cop-win reliability among all unicyclic and bicyclic graphs, respectively. While most of our methods are purely combinatorial, the analytic properties of the node cop-win reliability will also be used (and required) for some results. This is joint work with Maimoonah Ahmed.

**Melissa Huggan**, Vancouver Island University
*Pursuit-Evasion Games: New Results and Open Problems*

Friday
10:00-11:00

Pursuit-evasion games have been vastly studied over several decades by dozens of researchers worldwide. A central focus of this research has been on the game of Cops and Robbers. Recently, modifying the ruleset or parameter of interest has led to many intriguing new research directions. In this talk, we will examine a few game variants and new game parameters by highlighting past work, recent discoveries, and open problems. The goal of this talk is to provide several ideas for research collaboration.

**Trent Marbach**, Toronto Metropolitan University
*Pursuit-evasion games on latin square graphs*

Friday
11:30 - 12:00

In this talk, we present our recent investigation of various pursuit-evasion parameters on latin square graphs, including the cop number, metric dimension, and localization number. We will investigate upper and lower bounds of these properties for all latin square graphs, and also give particular latin square graphs that show when these bounds are close to being tight. Recent results on covers and partial transversals will assist us in this endeavor, giving a promising connection between fundamental questions in latin square theory and problems studied within the pursuit-evasion literature.

**John Marcoux**, Memorial University of Newfoundland
*Firefighting with a Distance-Based Restriction*

Friday
12:00 - 12:30

In the classic version of the game of firefighter, on the first turn a fire breaks out on a vertex in a graph $G$ and then $k$ firefighters protect $k$ vertices. On each subsequent turn, the fire spreads to the collective unburnt neighbourhood of all the burning vertices and the firefighters again protect $k$ vertices. Once a vertex has been burnt or protected it remains that way for the rest of the game. A common objective with respect to some infinite graph $G$ is to determine how many firefighters are necessary to stop the fire from spreading after a finite number of turns, commonly referred to as *containing* the fire. We introduce the concept of *distance-restricted firefighting* where the firefighters' movement is restricted so they can only move up to some fixed distance $d$ per turn rather than being able to move without restriction. We establish some general properties of this new game in contrast to properties of the original game, and we investigate specific cases of the distance-restricted game on the infinite square, strong, and hexagonal grids.

**Caleb Jones**, Memorial University of Newfoundland
*Extending Graph Burning to Hypergraphs*

Friday
2:00 - 2:30

We introduce a round-based model much like graph burning which applies to hypergraphs. The rules for this new model are very natural, and generalize the original model of graph burning. In particular, we obtain bounds on the burning number of a hypergraph. We show that arbitrary hypergraphs do not satisfy a bound analogous to the burning number conjecture, and we therefore investigate certain families of hypergraphs such as Steiner triple systems. The lazy burning model on hypergraphs is introduced, along with a new parameter, the lazy burning number. Interestingly, lazily burning a graph is trivial, while lazily burning a hypergraph can be quite complicated. Moreover, the lazy burning model is a useful tool for analyzing the round-based model on hypergraphs.

**Mohamed Omar**, Harvey Mudd College — Friday 2:30 - 3:00

*Burning Graph Classes*

Recent attention has been given to the process of graph burning, a process that models the propagation of nefarious agents through networks. Particular attention has been given to determining worst case analysis of the graph burning process. The Graph Burning Conjecture proposes an upper bound on the time a burning process takes to complete on a graph, as a function of the vertices. This upper bound has been verified for many graph classes across many papers, however many of the methods are particular to the given classes at hand. In contrast, we present systematic methods that take in a graph class and output graphs in the class that satisfy the conjecture. This approach hones in on some of the remaining mystery of the Graph Burning Conjecture, especially as it relates to graphs with prescribed degree conditions. This is joint work with Vibha Rohilla.

# Participants

**Anthony Bonato** (Toronto Metropolitan University)

**Boris Brimkov** (Slippery Rock University)

**Andrea Burgess** (University of New Brunswick Saint John)

**Ben Cameron** (The King's University)

**Nancy Clarke** (Acadia University)

**Danielle Cox** (Mount Saint Vincent University)

**Peter Danziger** (Toronto Metropolitan University)

**Thomas Dissaux** (INRIA Sophia Antipolis)

**Christopher Duffy** (University of Melbourne)

**Danny Dyer** (Memorial University of Newfoundland)

**Sean English** (University of Illinois-Urbana Champaign)

**Jessica Enright** (University of Glasgow)

**Joshua Erde** (Graz University of Technology)

**Gena Hahn** (Université de Montréal)

**Sam Hand** (University of Glasgow)

**Melissa Huggan** (Vancouver Island University)

**Svenja Huntemann** (Concordia University of Edmonton)

**Caleb Jones** (Memorial University of Newfoundland)

**Shahin Kamali** (York University)

**Bill Kinnersley** (University of Rhode Island)

**Narayanan Krishna** (Memorial University of Newfoundland)

**Gary MacGillivray** (University of Victoria)

**Kyle MacKeigan** (Mount Saint Vincent University)

**Evan MacTavish** (University of New Brunswick)

**Trent Marbach** (Toronto Metropolitan University)

**John Marcoux** (Memorial University of Newfoundland)

**Fionn McInerney** (CISPA Helmholtz Center for Information Security)

**Margaret-Ellen Messinger** (Mount Allison University)

**Bojan Mohar** (Simon Fraser University)

**Joy Morris** (University of Lethbridge)

**Todd Mullen** (Acadia University and St. Francis Xavier University)

**Kieka Mynhardt** (University of Victoria)

**Nicolas Nisse** (INRIA Sophia Antipolis)

**Kerry Ojakian** (CUNY Bronx)

**Mohamed Omar** (Harvey Mudd College)

**David Pike** (Memorial University of Newfoundland)

**Brittany Pittman** (Toronto Metropolitan University)

**Alyssa Sankey** (University of New Brunswick)

**Brendan Sullivan** (Emmanuel College)

**Florian Thomas** (University of Auckland and Graz University of Technology)

**Nikolas Townsend** (University of Rhode Island)

**Jérémie Turcotte** (McGill University)

**Christopher Van Bommel** (University of Manitoba)

**Virgelot Virgile** (University of Victoria)

**Ethan Williams** (University of Victoria)

**Petra Wolf** (Universitat Trier)

**Boting Yang** (University of Regina)

**Zhiyuan Zhang** (Toronto Metropolitan University)