New Variants and Open Problems in Graph Searching

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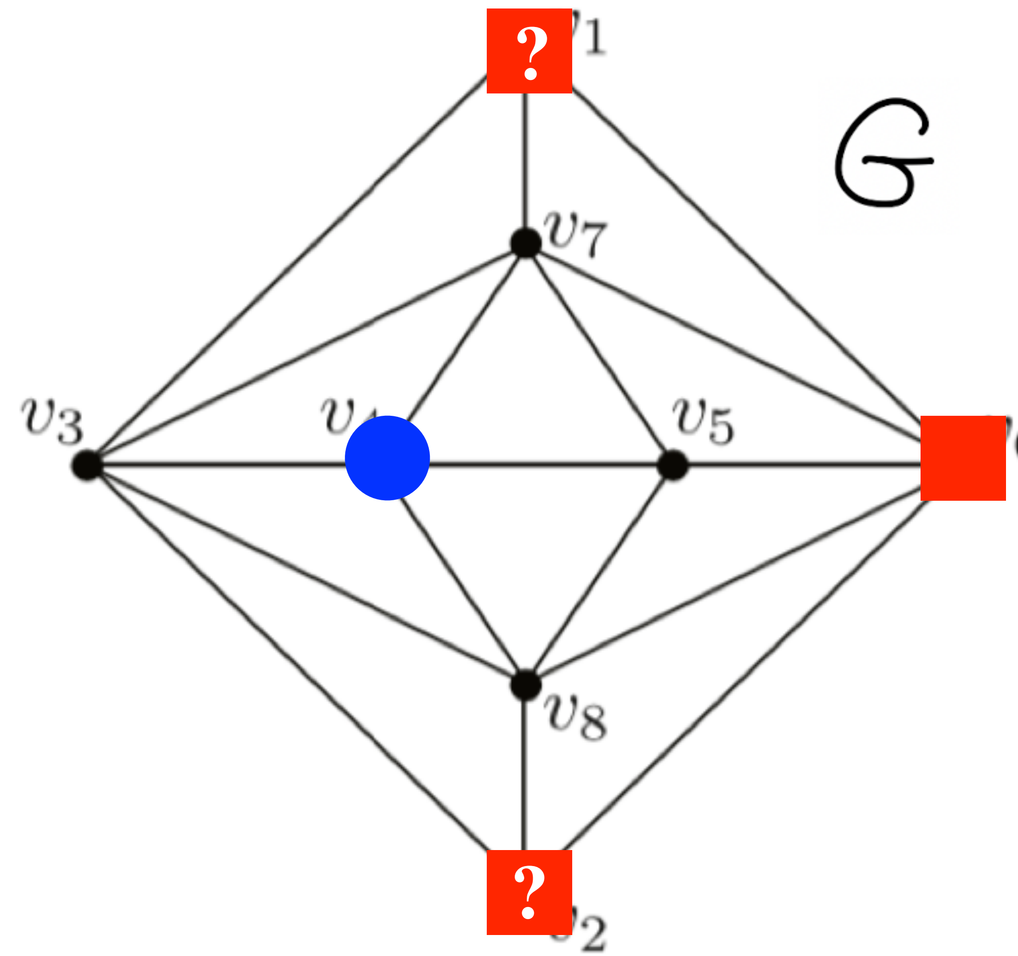
Limited Visibility Cops and Robber ($\ell \geq 0$)

N.E. Clarke, D. Cox, C. Duffy, D. Dyer, S.L. Fitzpatrick, M.E. Messinger, Limited Visibility Cops and Robber, Disc. Appl. Math. (2020)

⟶ built on work of Dereniowski, Dyer, Tifenbach, Yang (2015)

- at each step, a subset of the cops move and then the robber moves
- robber can always see the locations of the cops; cops can always see the locations of other cops
- all cops can see the robber's location of a cop is within distance ℓ of the robber



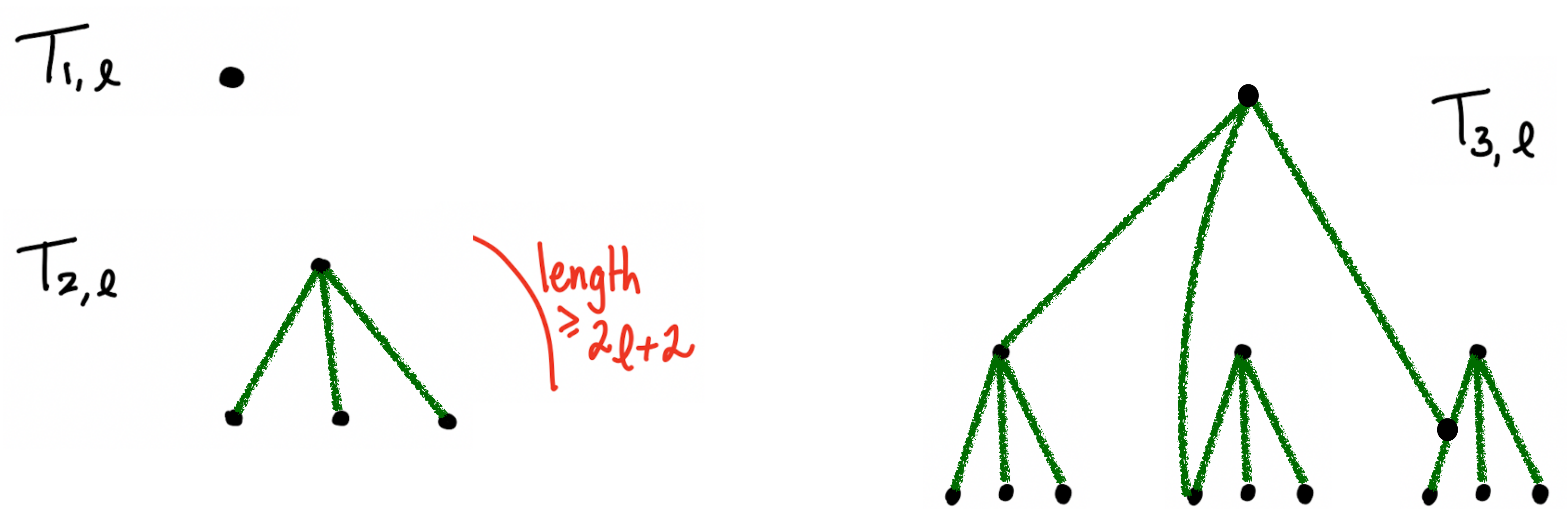
$C_\ell(G)$ = min # cops needed to capture R

$\ell = 1$

$C_1(G) = 2$

Limited Visibility Cops and Robber $(\ell \geq 0)$

Theorem 4.9. *If T is a tree, then $c_\ell(T) = k$, $\ell \geq 1$, where k the greatest integer such that T contains a subgraph from $\mathcal{T}_{k,\ell}$.*



Definition 4.2. For fixed $\ell \geq 1$, let $\mathcal{T}_{k,\ell}$ be the family of trees defined recursively as follows:

- $\mathcal{T}_{1,\ell} = \{K_1\}$
- $\mathcal{T}_{k,\ell}$ is the set of trees, T, that can be formed as follows: Let $T_1, T_2, T_3 \in \mathcal{T}_{k-1,\ell}$. Let r_1, r_2, r_3 be any vertices of T_1, T_2, T_3 respectively. The tree T is formed from the disjoint union of T_1, T_2, T_3, together with paths of length at least $2\ell + 2$ from each of r_1, r_2, r_3, to a common endpoint, q_k.

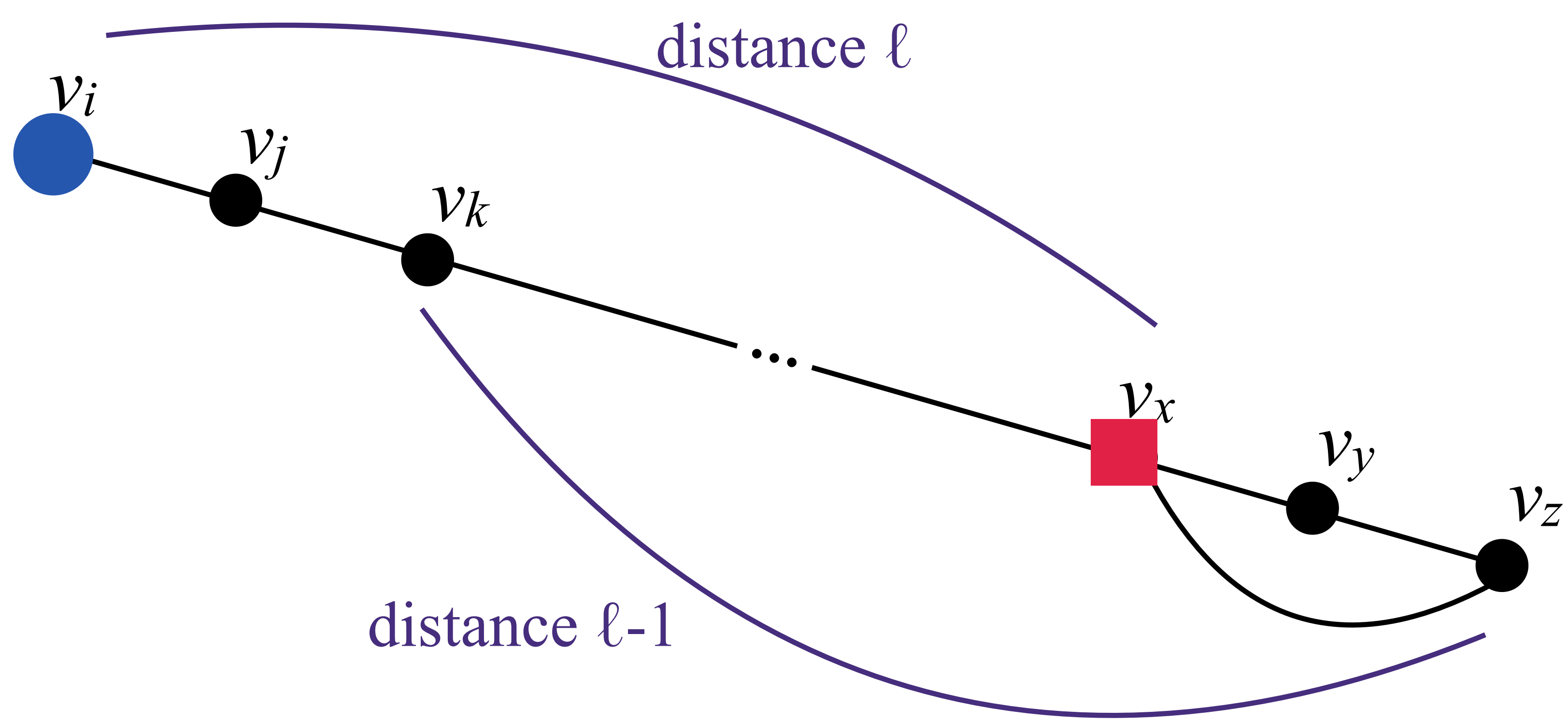
Limited Visibility Cops and Robber $(\ell \geq 0)$

$C_\ell(G)$ = min # cops needed to capture R

$C'_\ell(G)$ = min # cops needed to see R at some step

Theorem 3.1. *For any graph G and $\ell \geq 2$, either $c'_\ell(G) = c_\ell(G)$ or $c(G) \leq c_\ell(G) \leq c(G)+1$.*

Theorem 3.2. *If G is a chordal graph and $\ell \geq 0$, then $c'_\ell(G) = c_\ell(G)$. Furthermore, once the robber has been seen by a cop, that cop can capture the robber.*



$i < x$

$x < y$

$y > z$

Problem 5.2. Given a graph G, suppose that if an ℓ-visibility cop, C, is within distance ℓ of the robber in some round t, then C can then capture the robber in round t' for some $t' > t$, where the play of the other cops in rounds $t+1$ through t' is irrelevant. Characterize such graphs.

Problem 5.3. For each $\ell \geq 1$, is $c_\ell(G) - c'_\ell(G)$ bounded?

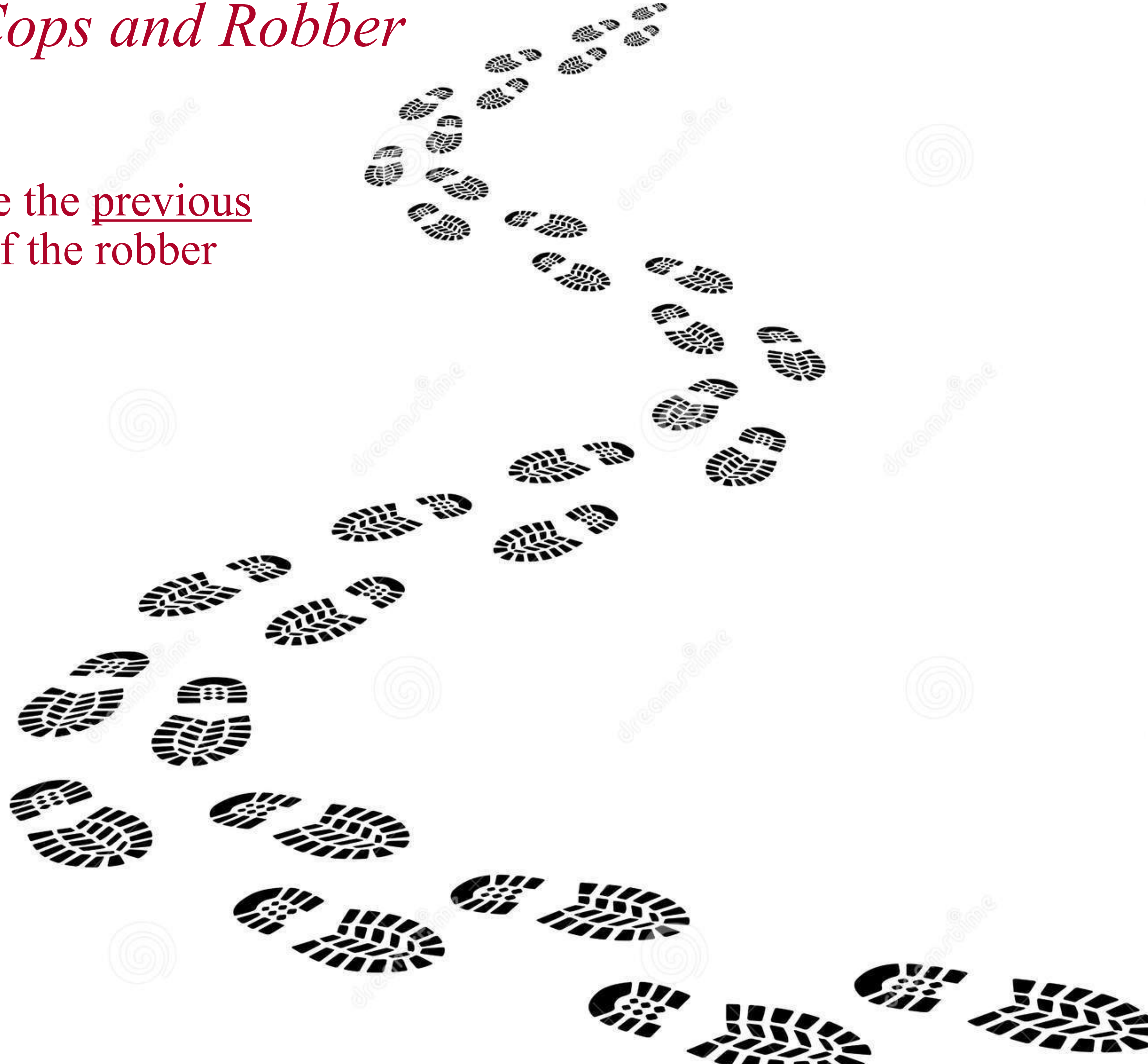
Problem 5.4. For $\ell \geq 1$, find a lower bound for $c'_\ell(G)$.

Problem 5.1. Provide a graph G for which $c_0(G) > c_1(G) > \cdots > c_{rad(G)}(G) > c(G)$.

Problem 5.5. What is the closure of $\frac{c_\ell(G)}{c_{\ell+1}(G)}$?

Time-Delay Cops and Robber

cops only see the <u>previous</u>
positions of the robber



Time-Delay Cops and Robber

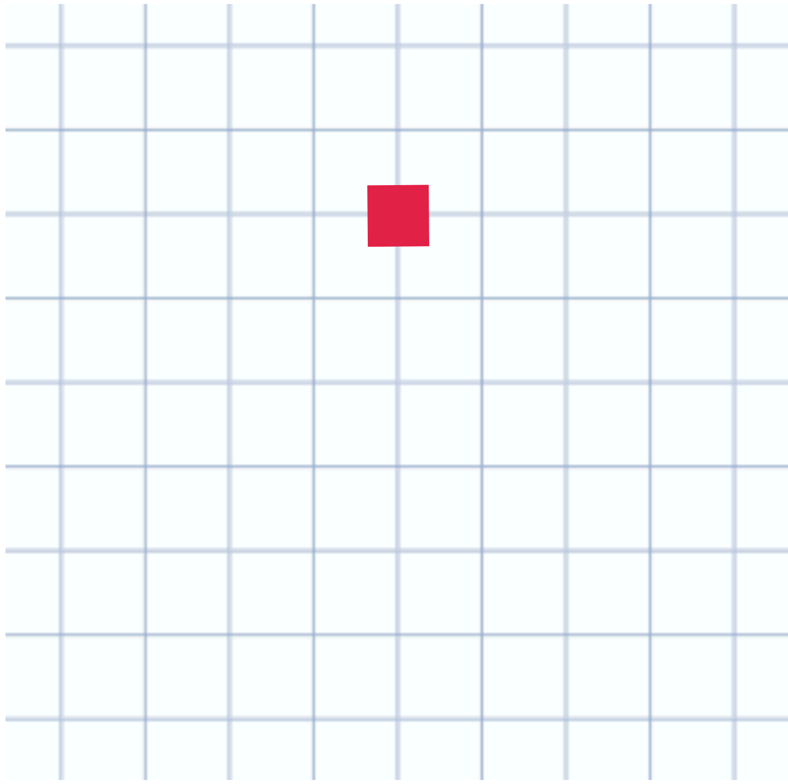
- cops only see the previous positions of the robber
- robber can always see the cops' positions
- each cop can always see the positions of the other cops

The Firefighter Problem

- fire initially breaks out at some vertex
- at each step,
  - ➤ firefighter protects some vertices

    then

  - ➤ fire spreads from each burned vertex to all unprotected (unburned) neighbors



Theorem (Wang & Moeller 2002): On the infinite Cartesian grid:

if 2 vertices are protected at each step, the fire can be contained in 8 steps.

if 1 vertex is protected at each step, the fire cannot be contained.

Theorem (Messinger 2008): On the infinite Cartesian grid, the fire can be contained by:

(Ng & Raff, 2008): no periodic sequence (to contain the fire) for which the average number of vertices protected at each step is less than 3/2.

(Feldheim & Hod, 2012): no periodic sequence (to contain the fire) for which the average number of vertices protected at each step is 3/2.

(Gavenčiak, Kratochvíl, Prałat, 2015): On the infinite Cartesian grid, the surviving rate is equal to 1/4.

The Pyro Game

at each step, the pyro chooses one burned vertex and spread from that vertex to all unprotected neighbours

The Pyro Game

Theorem (Messinger, Yarnell): On the infinite Cartesian grid, *one* firefighter can contain the fire.

Conjecture (Messinger, Yarnell): On the infinite Cartesian grid, *one* firefighter can prevent the pyro from burning any vertex distance 7 from the original burned vertex.

Question (Messinger, Yarnell): What is the minimum integer f such that f firefighters can contain the pyro on an infinite 3-dimensional grid?

The Pyro Game

<u>Theorem (Messinger, Yarnell):</u> On the infinite strong grid, *two* firefighters can contain the fire.

<u>Question (Messinger, Yarnell):</u> Can *one* firefighter contain the fire on the infinite strong grid?