Slides for talk "Product Throttling Power Domination" by Daniela Ferrero.

PRODUCT THROTTLING FOR POWER DOMINATION

Daniela Ferrero

Texas State University
San Marcos, TX

Joint work with:
Sarah E. Anderson, University of St. Thomas, St. Paul, MN
Karen E. Collins, Wesleyan University, Middletown, CT
Leslie Hogben, Iowa State University, Ames, IA
Carolyn Mayer, Sandia National Laboratories, Albuquerque, NM
Ann N. Trenk, Wellesley College, Wellesley, MA
Shanise Walker, University of Wisconsin, Eau Claire, WI

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Monitoring electrical power networks

- Electrical power networks must be continuously monitored to prevent blackouts and power surges, and in general, to guarantee good service.
- Monitoring an electrical power network means obtaining real time values of the state of the network at every location.
- A *Phasor Measurement Unit (PMU)* reads the state at the network location where it is placed, but placing a PMU at every location it is unfeasible and unnecessary.
- A set of PMUs are strategically placed at selected network locations and synchronized via GPS (Global Positioning System).
- The synchronized PMU readings are combined with Ohm's Laws, to determine the state of the network at every location without a PMU.

Monitoring electrical power networks

- A PMU placement is *feasible* if the PMU readings are sufficient to determine the state of the network at any location without a PMU.
- A feasible PMU placement is *optimal* if it uses the minimum number of PMUs.
- The *PMU Placement Problem* asks for an optimal PMU placement for a given power network.
- Teresa W. Haynes, Sandra M. Hedetniemi, Steve T. Hedetniemi & Michael A. Henning (2002) defined the *Power Domination Problem* in graph theory so that:

Electrical Engineering		Graph Theory
electrical power network	-	graph
feasible PMU placement	-	power dominating set
optimal PMU placement	-	minimum power dominating set
optimal number of PMUs	-	power domination number

- Dennis J. Brueni & Lenwood S. Heath (2005) simplified the definition of power domination and Ashkan Aazami (2008) introduced discrete time intervals.

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Bounds
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Power domination

Power dominating set

Let $G = (V, E)$ be a graph. Given $S \subseteq V$ recursively define $\{P^i(S)\}_{i \in \mathbb{Z}^+}$ by
1) $P^1(S) = N[S] = S \cup N(S)$.
2) $P^{i+1}(S) = P^i(S) \cup \{u \in V \setminus P^i(S) : \exists v \in P^i(S), N(v) \setminus P^i(S) = \{u\}\}$.

If there exists t such that $P^t(S) = V$ then S is a power dominating set of G.

Power propagation time of a set

The propagation time of a power dominating set S is
$$\mathrm{ppt}(S, G) = \min\{t \in \mathbb{Z}^+ : P^t(S) = V\}.$$

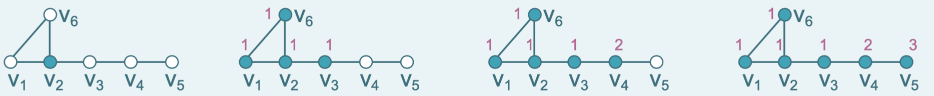
Power domination number of a graph

The power domination number of graph G is $\gamma_P(G)$ defined as
$$\gamma_P(G) = \min\{|S| : S \text{ power dominating set of } G\}.$$

Power propagation time of a graph

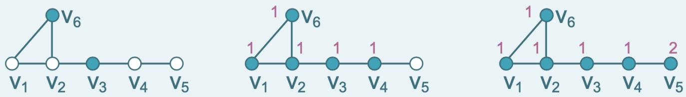
The propagation time of graph G is $\mathrm{ppt}(G)$ defined as
$$\mathrm{ppt}(G) = \min\{\mathrm{ppt}(G, S) : S \text{ power dominating set of } G \text{ and } |S| = \gamma_P(G)\}.$$

Examples



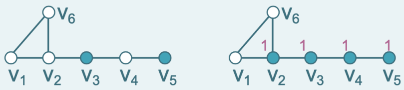
$S=\{v_2\}$ $P^1(S)=\{v_1,v_2,v_3,v_6\}$ $P^2(S)=\{v_1,v_2,v_3,v_4,v_6\}$ $P^3(S)=\{v_1,v_2,v_3,v_4,v_5,v_6\}$

S is a power dominating set and $\text{ppt}(S,G)=3$



$U=\{v_3,v_6\}$ $P^1(U)=\{v_1,v_2,v_3,v_4,v_6\}$ $P^2(U)=\{v_1,v_2,v_3,v_4,v_5,v_6\}$

U is power dominating set and $\text{ppt}(U,G)=2$



$T=\{v_3,v_5\}$ $P(T)=\{v_1,v_2,v_3,v_6\}$

★ Moreover, $|S|=1$ implies $\gamma_P(G)=1$
★ The only γ_P-sets of G are: $\{v_1\}$, $\{v_2\}$ and $\{v_6\}$

$\text{ppt}(\{v_1\},G)=4$
$\text{ppt}(\{v_2\},G)=3$
$\text{ppt}(\{v_6\},G)=4$
$\Big\}\ \text{ppt}(G)=3$

T is not a power dominating set

Back to electrical engineering

The deployment of large scale PMU systems started in 2010 and it has shown:

- In addition to preventing blackouts and power surges, data from PMUs helps to protect the power grid and enhance the service provided.
- The micro PMU (μPMU) offers a cost-effective alternative to a PMUs.
- Minimizing the number of PMUs produces unsatisfactory results, mainly due to the lack of redundancy to recover data losses due to unavoidable failures.
- A few additional PMUs produce enough advantages to offset the increased costs.

As a result:

- There is a practical interest in enhancing existing PMU systems by adding redundancy without a major increase on the number of PMUs.
- In terms of power domination, this new problem translates into studying the compromise between the cardinality of a power dominating set and its propagation time.
- In particular, minimizing the product of the cardinality of a power dominating set and its propagation time provides an important insight for electrical engineers.

Product throttling for γ_P

Power product throttling number of a set

Let S be a power dominating set of a graph G. The product power throttling number of S in G is $\text{th}^{\times}_{\gamma_P}(G,S)$ defined by $\text{th}^{\times}_{\gamma_P}(G,S) = |S|\,\text{ppt}(G,S)$.

Power product throttling number of a graph

The product power throttling number of a graph G is $\text{th}^{\times}_{\gamma_P}(G)$ defined by $\text{th}^{\times}_{\gamma_P}(G) = \min\{\text{th}^{\times}_{\gamma_P}(S,G) : S$ power dominating set of $G\}$.

Remarks:

- Throttling was a term previously used in other forms of graph searching for the problem of minimizing either the sum or the product, of the number of searchers and the search time.
- Product throttling for power domination was first proposed and studied by Sarah E. Anderson, Karen L. Collins, Leslie Hogben, Carolyn Mayer, Ann N. Trenk, Shanise Walker and myself. This talk contains highlights of our results.

Bounds on $\text{th}^{\times}_{\gamma_P}(G)$

By definition:

- $\text{th}^{\times}_{\gamma_P}(G) \le \gamma(G)$, the domination number of G
- $\gamma_P(G) \le \text{th}^{\times}_{\gamma_P}(G)$

Consequences:

- $\text{th}^{\times}_{\gamma_P}(G) = \min\{\text{th}^{\times}_{\gamma_P}(S,G) : S$ power dominating set of $G, \gamma_P(G) \le |S| \le \gamma(G)\}$
- If $\gamma(G) = 1$ then $\text{th}^{\times}_{\gamma_P}(G) = 1$; moreover, $\gamma(G) = 1$ is equivalent to $\text{th}^{\times}_{\gamma_P}(G) = 1$

Graph families:

- $\text{th}^{\times}_{\gamma_P}(K_n) = 1$, where K_n is the complete graph of order $n \ge 2$
- $\text{th}^{\times}_{\gamma_P}(K_{1,n}) = 1$, where $K_{1,n}$ is the star of order $n+1$ for $n \ge 0$
- $\text{th}^{\times}_{\gamma_P}(W_n) = 1$, where W_n is the wheel of order $n+1$ $n \ge 3$

Bounds for $\text{th}_{\gamma_P}^{\times}(G)$

Theorem (DF, Hogben, Kenter, Young, 2017)

In a connected graph G with maximum degree Δ, $\gamma_P(G) \geq \frac{|G|}{\text{ppt}(G)\Delta+1}$.

Corollary (Anderson, Collins, DF, Hogben, Mayer, Trenk, Walker, 2020)

In a connected graph G with maximum degree Δ, $\text{th}_{\gamma_P}^{\times}(G) \geq \left\lceil \frac{|G|}{\Delta+1} \right\rceil$.

Consequence: If a connected graph G has maximum degree Δ and domination number $\gamma(G) = \left\lceil \frac{|G|}{\Delta+1} \right\rceil$ then $\text{th}_{\gamma_P}^{\times}(G) = \left\lceil \frac{|G|}{\Delta+1} \right\rceil$.

Graph families:

- $\text{th}_{\gamma_P}^{\times}(P_n) = \left\lceil \frac{n}{3} \right\rceil$, where P_n is the path of order $n \geq 2$
- $\text{th}_{\gamma_P}^{\times}(C_n) = \left\lceil \frac{n}{3} \right\rceil$, where C_n is the cycle of order $n \geq 3$

Domination upper bound

Lemma (Anderson, Collins, DF, Hogben, Mayer, Trenk, Walker, 2019)

Let G be a connected graph and let b be a positive integer.
If $\gamma(G) \leq b$ and $\gamma_P(G) \geq \frac{b}{2}$, then $\text{th}_{\gamma_P}^{\times}(G) = \gamma(G)$.

Proof: Let S be a power dominating set of G with $\gamma_P(G) \leq |S| \leq \gamma(G)$.

- If $|S| = \gamma(G)$, then $\text{ppt}(G,S) = 1$ and $\text{th}_{\gamma_P}^{\times}(G,S) = \gamma(G)$.
- If $|S| < \gamma(G)$, then $\text{ppt}(G,S) \geq 2$ and $\text{th}_{\gamma_P}^{\times}(G,S) \geq 2\gamma_P(G)$.

From $\gamma_P(G) \geq \frac{b}{2}$ follows $\text{th}_{\gamma_P}^{\times}(G,S) \geq b \geq \gamma(G)$ and thus, $\text{th}_{\gamma_P}^{\times}(G) = \gamma(G)$. □

It is well-known that in a connected graph G, $\gamma(G) \leq \frac{|G|}{2}$. Combining this result, other known upper bounds for $\gamma(G)$ and the Lemma above:

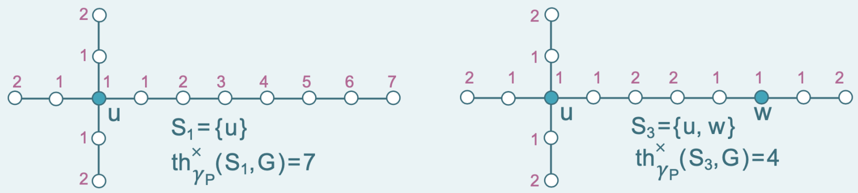
Corollary (Anderson, Collins, DF, Hogben, Mayer, Trenk, Walker, 2020)

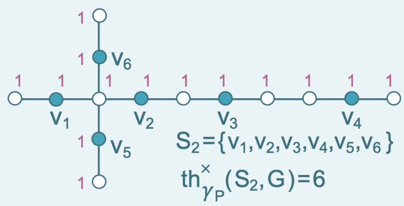
Let G be a connected graph of order $n \geq 2$ and minimum degree δ.

- *If $\gamma_P(G) \geq \frac{n}{4}$, then $\text{th}_{\gamma_P}^{\times}(G) = \gamma(G)$.*
- *If $\delta \geq 2$ and $\gamma_P(G) \geq \frac{n}{5}$, then $\text{th}_{\gamma_P}^{\times}(G) = \gamma(G)$.*
- *If $\delta \geq 3$ and $\gamma_P(G) \leq \frac{3n}{16}$, then $\text{th}_{\gamma_P}^{\times}(G) = \gamma(G)$.*

Example

A graph G with $\gamma(G) > \text{th}^{\times}_{\gamma_P}(G) > \gamma_P(G)$.





$\text{th}^{\times}_{\gamma_P}(G)=4$ $\gamma_P(G)=1$
$\gamma(G)=6$ $\text{ppt}(G)=7$

Maximum value of $\text{th}^{\times}_{\gamma_P}(G)$

The domination upper bound implies that in any connected graph G, $\text{th}^{\times}_{\gamma_P}(G) \leq \frac{|G|}{2}$. Using next result, we characterized all connected graphs G with $\text{th}^{\times}_{\gamma_P}(G) = \frac{|G|}{2}$.

Theorem (Payan, Xuong, 1982 or Fink, Jacobson, Kinch, J. Roberts, 1985)

A connected graph G has $\gamma(G) = \frac{|G|}{2}$ if and only if $G = H \circ K_1$ for some connected graph H or $G = C_4$.

Recall: Graph $H \circ K_1$ is the corona of graphs H and K_1 obtained by adding a neighbor of degree 1 to each vertex of H.

Theorem (Anderson, Collins, DF, Hogben, Mayer, Trenk, Walker, 2020)

A connected graph G has $\text{th}^{\times}_{\gamma_P}(G) = \frac{|G|}{2}$ if and only if $G = (H \circ K_1) \circ K_1$ for some connected graph H, $G = C_4 \circ K_1$, or $G = C_4$.

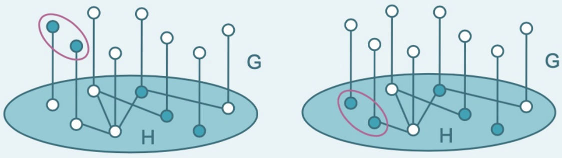
Maximum value of $\mathrm{th}_{\gamma_P}^{\times}(G)$

Theorem (Anderson, Collins, DF, Hogben, Mayer, Trenk, Walker, 2020)

A connected graph G has $\mathrm{th}_{\gamma_P}^{\times}(G) = \frac{|G|}{2}$ if and only if $G = (H \circ K_1) \circ K_1$ for some connected graph H, $G = C_4 \circ K_1$, or $G = C_4$.

Idea of the proof:

1. If S is a PDS of $H \circ K_1$ not contained in H, there exists T, PDS of $H \circ K_1$ contained in H with $\mathrm{th}_{\gamma_P}^{\times}(H \circ K_1, T) \leq \mathrm{th}_{\gamma_P}^{\times}(H \circ K_1, S)$.



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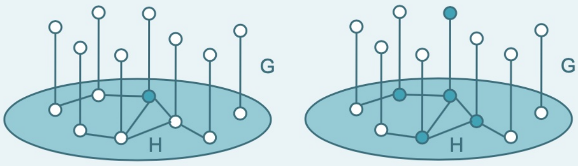
Maximum value of $\mathrm{th}_{\gamma_P}^{\times}(G)$

Theorem (Anderson, Collins, DF, Hogben, Mayer, Trenk, Walker, 2020)

A connected graph G has $\mathrm{th}_{\gamma_P}^{\times}(G) = \frac{|G|}{2}$ if and only if $G = (H \circ K_1) \circ K_1$ for some connected graph H, $G = C_4 \circ K_1$, or $G = C_4$.

Idea of the proof:

1. If S is a PDS of $H \circ K_1$ not contained in H, there exists T, PDS of $H \circ K_1$ contained in H with $\mathrm{th}_{\gamma_P}^{\times}(H \circ K_1, T) \leq \mathrm{th}_{\gamma_P}^{\times}(H \circ K_1, S)$.
2. If T is a power dominating set of $H \circ K_1$ containing only vertices of H and $\mathrm{ppt}(H \circ K_1, T) \geq 2$, then T is a dominating set of H.



Maximum value of $\text{th}_{\gamma_P}^{\times}(G)$

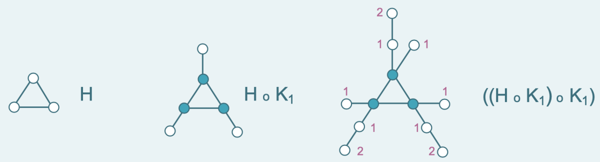
Theorem (Anderson, Collins, DF, Hogben, Mayer, Trenk, Walker, 2020)

A connected graph has G $\text{th}_{\gamma_P}^{\times}(G) = \frac{|G|}{2}$ if and only if $G = (H \circ K_1) \circ K_1$ for some connected graph H, $G = C_4 \circ K_1$, or $G = C_4$.

Idea of the proof:

1. If S is a PDS of $H \circ K_1$ not contained in H, there exists T, PDS of $H \circ K_1$ contained in H with $\text{th}_{\gamma_P}^{\times}(H \circ K_1, T) \leq \text{th}_{\gamma_P}^{\times}(H \circ K_1, S)$.
2. If T is a power dominating set of $H \circ K_1$ containing only vertices of H and $\text{ppt}(H \circ K_1, T) \geq 2$, then T must be a dominating set of H.
3. Then, $\text{th}_{\gamma_P}^{\times}(H \circ K_1) = 2\gamma(H)$ and $\text{th}_{\gamma_P}^{\times}((H \circ K_1) \circ K_1) = 2\gamma(H \circ K_1)$.
4. We know $\gamma(H \circ K_1) = \frac{|H \circ K_1|}{2} = |H|$ and $\gamma(C_4)$=2.
5. From 3 and 4: $\text{th}_{\gamma_P}^{\times}((H \circ K_1) \circ K_1) = 2|H| = \frac{|(H \circ K_1) \circ K_1|}{2}$ and $\text{th}_{\gamma_P}^{\times}(C_4 \circ K_1) = 4$. □

Example:

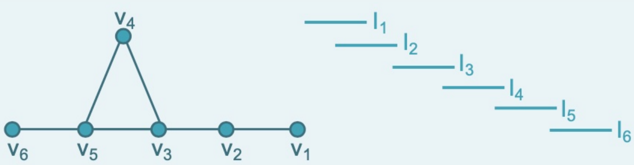


Unit interval graphs

Unit interval graph

A graph G is a unit interval graph if each vertex v of G can be assigned a closed unit length real interval $I(v)$ so that vertices are adjacent if and only if their assigned intervals intersect.

Example:

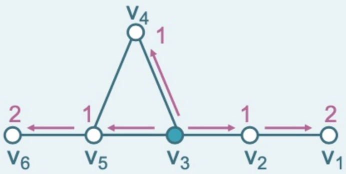


Remarks:

- Any unit interval graph has a representation with intervals of different endpoints, which induces an order in the vertex set.
- In this order, the closed neighborhood of any vertex is a set of consecutive vertices. For example, in the graph above:
  $$v_1 < v_2 < v_3 < v_4 < v_5 < v_6,\ N[v_5] = \{v_3, v_4, v_5, v_6\}.$$

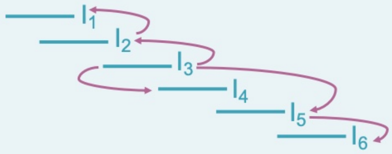
Unit interval graphs

The previous properties have important consequence for the study of power domination in unit interval graphs.



v_3, v_2, v_1
v_3, v_4
v_3, v_5, v_6

The propagation sequences respect the order induced by the intervals, or its reverse.



v_3, v_2, v_1
v_3, v_4
v_3, v_5, v_6

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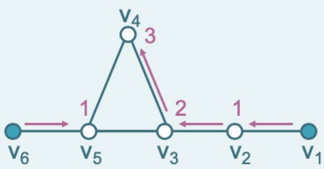
Unit interval graphs

The same occurs with a power dominating set that is not minimum.



v_1, v_2, v_3, v_4
v_6, v_5

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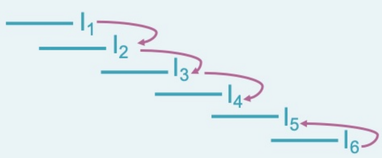

v_1, v_2, v_3, v_4
v_6, v_5

Unit interval graphs

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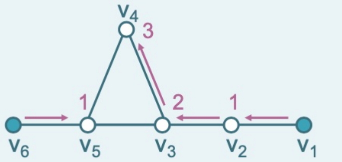
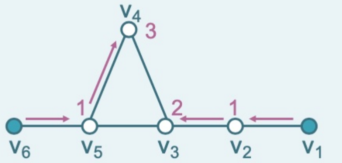


v_1, v_2, v_3, v_4
v_6, v_5

Even if we choose propagation sequences in a different way.



v_1, v_2, v_3
v_6, v_5, v_4

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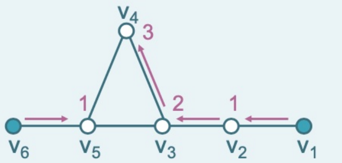

v_1, v_2, v_3, v_4
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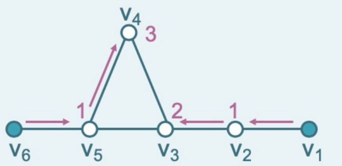


v_1, v_2, v_3
v_6, v_5, v_4

Unit interval graphs

Why? Consecutive vertices in a propagation sequence must be neighbors and in a connected unit interval graph for every vertex v, $N[v]$ is a set of consecutive vertices.

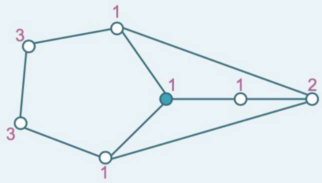
This observation has two consequences:

Lemma (Anderson, Collins, Ferrero, Hogben, Mayer, Trenk, Walker, 2020)

Let G be a connected unit interval graph with power dominating set S. Then, $|P^k(S) \setminus P^{k-1}(S)| \leq 2|S|$ *for every* $2 \leq k \leq \text{ppt}(G, S)$.

For an arbitrary graph G, a vertex power dominated at time k does not need to have as neighbor power dominated at time $k-1$

Example:



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For an arbitrary graph G, a vertex power dominated at time k does not need to have as neighbor power dominated at time $k-1$, but this happens in a unit interval graph:

Lemma (Anderson, Collins, Ferrero, Hogben, Mayer, Trenk, Walker, 2020)

If G is a connected unit interval graph with power dominating set S, then for each $k \geq 3$, every vertex in $P^k(S) \setminus P^{k-1}(S)$ is adjacent to a vertex in $P^{(k-1)}(S) \setminus P^{k-2}(S)$.

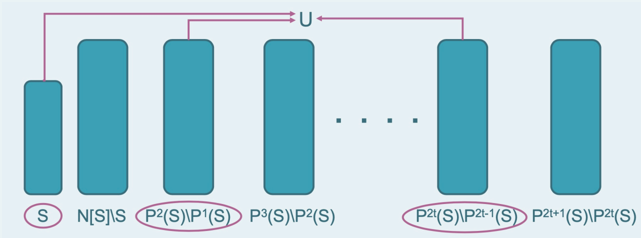
Unit interval graphs

Theorem (Anderson, Collins, Ferrero, Hogben, Mayer, Trenk, Walker, 2020)

If G is a connected unit interval graph, then $\text{th}_{\gamma_P}^{\times}(G) = \gamma(G)$.

Idea of the proof:

- Assume $\text{th}_{\gamma_P}^{\times}(G) = \text{th}_{\gamma_P}^{\times}(G,S) = |S|\,\text{ppt}(G,S)$.
- If $\text{ppt}(G,S) = 1$ then S is a dominating set and $\text{th}_{\gamma_P}^{\times}(G) = \gamma(G)$.
- If not, let t be an integer such that $\text{ppt}(G,S) = 2t$ or $\text{ppt}(G,S) = 2t+1$.
- If $\text{ppt}(G,S) = 2t+1$: Let $U = S \cup_{j=1}^{t} P^{2j}(S) \setminus P^{2j-1}(S)$.



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Idea of the proof:

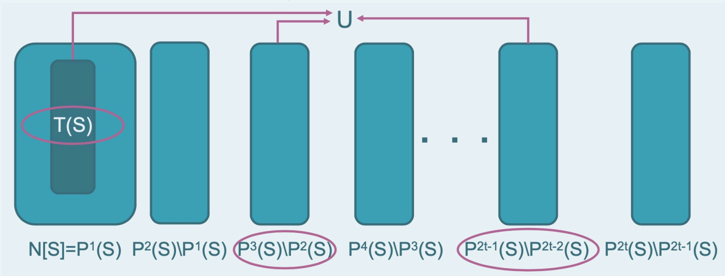
- Assume $\text{th}_{\gamma_P}^{\times}(G) = \text{th}_{\gamma_P}^{\times}(G,S) = |S|\,\text{ppt}(G,S)$.
- If $\text{ppt}(G,S) = 1$ then S is a dominating set and $\text{th}_{\gamma_P}^{\times}(G) = \gamma(G)$.
- Let t be an integer such that $\text{ppt}(G,S) = 2t$ or $\text{ppt}(G,S) = 2t+1$.
- If $\text{ppt}(G,S) = 2t+1$: Let $U = S \cup_{j=1}^{t} P^{2j}(S) \setminus P^{2j-1}(S)$. Then
  $|U| \leq |S| + \sum_{j=1}^{t} |P^{2j}(S) \setminus P^{2j-1}(S)| \leq |S| + 2t|S| = \text{ppt}(G,S)|S|$.
- We show U is a dominating set of G to conclude $\gamma(G) \leq |U| \leq \text{ppt}(G,S)|S|$.
- This is sufficient to show $\gamma(G) \leq |S|\,\text{ppt}(G,S)$, since
  $|S|\,\text{ppt}(G,S) = \text{th}_{\gamma_P}^{\times}(G) \leq \gamma(G)$ follows from G being connected.

Unit interval graphs

Idea of the proof (cont.):

The remaining case is when $\mathrm{ppt}(G, S)$ is even.

- Assume $S = \{s_1, s_2, \ldots, s_p\}$ where $s_1 < s_2 < \cdots < s_p$.
  Let u_i (v_i) denote the least (greatest) neighbor of s_i.
  Let $T(S) = \{u_1, u_2, \ldots, u_p\} \cup \{v_1, v_2, \ldots, v_p\}$.
- Prove $T(S) \subseteq P^1(S) \subset P^2(S)$ is a dominating set of $P^2(S)$.
- If $\mathrm{ppt}(G, S) = 2t$, let $U = T(S) \cup_{j=1}^{t-1} P^{2j+1}(S) \setminus P^j(S)$.



Unit interval graphs

Idea of the proof (cont.):

The remaining case is when $\mathrm{ppt}(G, S)$ is even.

- Assume $S = \{s_1, s_2, \ldots, s_p\}$ where $s_1 < s_2 < \cdots < s_p$.
  Let u_i (v_i) denote the least (greatest) neighbor of s_i.
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- Prove $T(S) \subseteq P^1(S) \subset P^2(S)$ is a dominating set of $P^2(S)$.
- If $\mathrm{ppt}(G, S) = 2t$, let $U = T(S) \cup_{j=1}^{t-1} P^{2j+1}(S) \setminus P^j(S)$.
  $|U| \leq |T(S)| + \sum_{j=1}^{t-1} |P^{2j}(S) \setminus P^{2j-1}(S)| \leq |2t|S| = \mathrm{ppt}(G, S)|S|$
- We show U is a dominating set of G to conclude $\gamma(G) \leq |U| \leq \mathrm{ppt}(G, S)|S|$.
- This is sufficient to show $\gamma(G) \leq |S|\,\mathrm{ppt}(G, S)$, since
  $|S|\,\mathrm{ppt}(G, S) = \mathrm{th}_{\gamma_P}^{\times}(G) \leq \gamma(G)$ follows from G being connected. $\square$

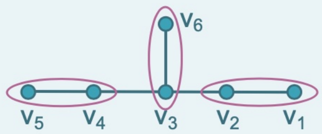
Unit interval graphs

Theorem (Anderson, Collins, Ferrero, Hogben, Mayer, Trenk, Walker, 2020)

If G is a connected unit interval graph, then $\mathrm{th}_{\gamma_P}^{\times}(G) = \gamma(G)$.

Observations:

- For an interval graph G, $\gamma(G)$ can be computed in polynomial time.
- For $\mathrm{th}_{\gamma_P}^{\times}(G) = \gamma(G)$ it is necessary for G to be a *unit* interval graph:



The interval graph G above has $\mathrm{th}_{\gamma_P}^{\times}(G) \leq \mathrm{th}_{\gamma_P}^{\times}(G, \{v_3\}) = 2$ and $\gamma(G) \geq 3$ because a dominating set must have one endpoint of each of the circled edges.

Problem:

Product throttling power domination in interval graphs that are not unit interval graphs.

Introduction to Power Domination
Throttling Power Domination
Bounds
Unit Interval Graphs

The end

Work presented in this talk

- S.E. Anderson, K.L. Collins, D. Ferrero, L. Hogben, C. Mayer, A.N. Trenk, S. Walker. Product throttling for power domination. Under review. arXiv:2010.16315 [math.CO].
- D. Ferrero, L. Hogben, F.H.J. Kenter, M. Young. Note on power propagation time and lower bounds for the power domination number. J. Comb. Optim. 34, 736-741, 2017.

Also in this area

- S.E. Anderson, K.L. Collins, D. Ferrero, L. Hogben, C. Mayer, A.N. Trenk, S. Walker. Survey of product throttling. arXiv:2012.12807 [math.CO].

  Chapter 2 in: Ferrero, D., Hogben, L., Kingan, S., Matthews, G.L. (Eds.) Research Trends in Graph Theory and Applications. Association for Women in Mathematics Series, Vol. XXV, Springer, Cham.

Thank you so much for listening to this talk!