8th Graph Searching in Canada (GRASCan) Workshop

August 6–7, 2019

The Fields Institute

Toronto, Canada

Ryerson University

FIELDS

• All talks will take place at The Fields Institute for Research in Mathematical Sciences, in the Stewart Library.
• The Stewart Library is equipped with all audiovisual equipment.
• The Stewart Library and Room 210 are available for participants to use in the afternoons.

Tuesday, August 6

8:45 - 9:00	Welcome
9:00 - 10:00	**Leslie Hogben** *Propagation and throttling for zero forcing, power domination, & Cops and Robbers*
10:00 - 10:30	COFFEE
10:30 - 11:00	**Andrea Burgess** *Cops that surround a robber*
11:00 - 11:30	**Boris Brimkov** *Catching a robber quickly with few cops*
11:30 - 12:00	**Jesse Geneson** *Throttling for the Cop versus Robber Game*
12:00 - 12:30	**Melissa Huggan** *Humans Capturing Robots*

Wednesday, August 7

9:00 - 10:00	**Boting Yang**, University of Regina *Fast Searching on Graphs*
10:00 - 10:30	COFFEE
10:30 - 11:00	**Stephen Finbow** *Firefighter problem on planar graphs*
11:00 - 11:30	**David Pike** *The Firebreak Problem*
11:30 - 12:00	**Shahin Kamali** *On complexity of burning and broadcasting problems*
12:00 - 12:30	**Fionn Mc Inerney** *Sequential Metric Dimension*

Abstracts of Talks

Plenary Talks

Leslie Hogben, Iowa State University and American Institute of Mathematics — Tuesday
Propagation and throttling for zero forcing, power domination, and Cops and Robbers — 9:00 - 10:00

The graph processes of power domination and zero forcing are involved in the analysis of electrical network monitoring and other applications, and pursuit-evasion games such as Cops and Robbers can be used to analyze intrusion detection; all contain an element of graph searching. For each of these parameters, the amount of time that it takes to finish the process (time cost, also known as propagation or capture time) is also studied. The cost trade-off between time and resources when the process uses more than the minimum possible number of resources is called throttling. This talk will survey recent results on propagation and throttling for these topics.

Boting Yang, University of Regina — Wednesday
Fast Searching on Graphs — 9:00 - 10:00

Given a graph, suppose that a fugitive hides on vertices or along edges of the graph. The fast searching problem is to find the minimum number of searchers required to capture the fugitive satisfying the constraint that every edge is traversed exactly once and searchers are not allowed to jump. In this talk, we present results on the fast search number of the Cartesian product of graphs, including hypercubes and toroidal grids. We give lower bounds and upper bounds on the fast search number of complete k-partite graphs. We also present results on the fast search number of other classes of graphs, such as Halin graphs, cubic graphs, bipartite graphs and split graphs.

Contributed Talks

Boris Brimkov, Rice University | Tuesday
Catching a robber quickly with few cops | 11:00 - 11:30

Cops and Robbers is a pursuit-evasion game played on a graph, between a team of cops and a single robber. The cops and the robber take turns moving to adjacent vertices; the goal for the cops is to capture the robber by occupying the same vertex as the robber, and the goal of the robber is to avoid capture. We consider the cop-throttling number of a graph G, which is defined as the minimum possible value of (k + the minimum number of turns needed for k cops to capture the robber on G over all possible games). This talk will provide some tools for computing and bounding the cop-throttling number, and relate it to other graph parameters.

Andrea Burgess, UNB SJ | Tuesday
Cops that surround a robber | 10:30 - 11:00

We introduce a variation on the traditional cops and robbers game where the cops win the game by occupying each of the robber's neighbours. The surrounding cop number $s(G)$ of a graph G is the minimum number of cops needed to guarantee that the robber will be caught (surrounded) in a finite number of moves. An easy observation is that cop number $c(G)$ is at most $s(G)$ since, in the original game, if the robber is surrounded then they would be captured in the next move.

In general, Meyniel's conjecture is far from true for $s(G)$. For example, $s(K_n) = n-1$, $s(K_{n,m}) = \min(n, m)$, and more generally $s(G) \geq \delta$. However, $s(G)$ is close to $c(G)$ for graphs such as the generalised Petersen graph or the incidence graph of a projective plane. Our initial findings also include results for the behaviour of the surrounding cop number under various graph products.

Stephen Finbow, St. Francis Xavier University | Wednesday
Firefighter problem on planar graphs | 10:30 - 11:00

Suppose that a fire breaks out at a vertex v of a graph G. As time passes, a firefighter protects vertices not yet on fire and the fire spreads to all the unprotected vertices that have a neighbour on fire. The k-surviving rate of G, denoted $\rho_k(G)$ is defined to be the expected proportion of saved vertices when the initial vertex on fire is chosen uniformly, at random. The question of finding a positive bound for the 2-surviving rate of planar graphs will be discussed.

Jesse Geneson, Iowa State University | Tuesday
Throttling for the Cop versus Robber Game | 11:30 - 12:00

Meyniel conjectured that the number of cops required to win the cop versus robber game on any connected n-vertex graph is at most a multiple of the square root of n. Despite the fact that several families of connected graphs are known to have cop number at least a multiple of the square root of n and none are known to have higher cop number, the best current upper bound on the cop number of n-vertex graphs is only slightly sub-linear.

The cop-throttling number of a graph sums the number of cops with the capture time (allowing more cops than the minimal number that can win the graph). Like the cop number, the cop-throttling number was conjectured to be at most a multiple of the square root of n for any connected n-vertex graph. We discuss a family of graphs that refute this conjecture, along with bounds on cop-throttling for trees and other families.

Melissa Huggan, Dalhousie University — Tuesday 12:00 - 12:30
Humans Capturing Robots

Robots have fast reactions and they can react to a human move even when moving simultaneously. (Witness the robot that has a 100% record against humans at Rock, Paper Scissors.) We look at this scenario in the context of Cops and Robbers. (Guest appearance by Richard Nowakowski, who will play the role of the cheating robot.)

Shahin Kamali, University of Manitoba — Wednesday 11:30 - 12:00
On complexity of burning and broadcasting problems

Given an unweighted, undirected graph, there are many broadcasting and burning protocols for dissemination of information in (or burning of) the graph. We review some of these protocols from an algorithmic point of view. The focus of the talk will be on telephone broadcasting, firefighter problem, and graph burning problem. Finding optimal dissemination schemes is NP-hard in all these problems. Despite similarities like this, these problems have different behaviour with respect to approximation algorithms. We review a recently-introduced algorithm for the graph burning problem which has a constant approximation factor. Meanwhile, the existing hardness result indicate that fire-fighter problem is APX-hard. For telephone broadcasting, the presence of an approximation algorithm with constant factor is an open problem. We review the existing approximation algorithms for telephone broadcasting and pose a few open problems relating the burning and broadcasting problems.

Fionn Mc Inerney, Ryerson University — Wednesday 12:00 - 12:30
Sequential Metric Dimension

In the localization game, introduced by Seager in 2013, an invisible and immobile target is hidden at some vertex of a graph G. At every step, one vertex v of G can be probed which results in the knowledge of the distance between v and the secret location of the target. The objective of the game is to minimize the number of steps needed to locate the target whatever be its location.

We address the generalization of this game where $k \geq 1$ vertices can be probed at every step. Our game also generalizes the notion of the *metric dimension* of a graph. Precisely, given a graph G and two integers $k, \ell \geq 1$, the LOCALIZATION problem asks whether there exists a strategy to locate a target hidden in G in at most ℓ steps and probing at most k vertices per step. We first show that, in general, this problem is NP-complete for every fixed $k \geq 1$ (respectively, $\ell \geq 1$). We then focus on the class of trees. On the negative side, we prove that the LOCALIZATION problem is NP-complete in trees when k and ℓ are part of the input. On the positive side, we design a (+1)-approximation algorithm for the problem in n-node trees; that is, an algorithm that computes in time $O(n \log n)$ (independent of k) a strategy to locate the target in at most one more step than an optimal strategy. This algorithm can be used to solve the LOCALIZATION problem in trees in polynomial time if k is fixed.

We also consider some of these questions in the context where, upon probing the vertices, the relative distances to the target are retrieved. This variant of the problem generalizes the notion of the *centroidal dimension* of a graph.

David Pike, Memorial University Wednesday
The Firebreak Problem 11:00 - 11:30

Suppose we have a network that is represented by a graph G. Potentially a fire (or other type of contagion) might erupt at some vertex of G. We are able to respond to this outbreak by establishing a firebreak at k other vertices of G, so that the fire cannot pass through these fortified vertices. The question that now arises is which k vertices will result in the greatest number of vertices being saved from the fire, assuming that the fire will spread to every vertex that is not fully behind the k vertices of the firebreak. This is the essence of the Firebreak decision problem, which we establish is intractable on the class of split graphs as well as on the class of bipartite graphs, but can be solved in linear time when restricted to graphs having constant-bounded treewidth, or in polynomial time when restricted to intersection graphs.

Participants

Anthony Bonato (Ryerson University)

Peter Bradshaw (Simon Fraser University)

Boris Brimkov (Rice University)

Andrea Burgess (UNB Saint John)

Nancy Clarke (Acadia University)

Danielle Cox (Mount Saint Vincent University)

Iren Darijani (Memorial University)

Jessica Enright (University of Edinburgh)

Stephen Finbow (St. Francis Xavier University)

Shannon Fitzpatrick (University of Prince Edward Island)

Jesse Geneson (Iowa State University)

Leslie Hogben (Iowa State University)

Saeed Hosseini (Simon Fraser University)

Melissa Huggan (Dalhousie University)

Shahin Kamali (University of Manitoba)

Bill Kay (Ryerson University)

Bill Kinnersley (University of Rhode Island)

Rehan Malik (Ryerson University)

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