Graph Searching in America (GRASAm) Workshop (11th in Series)

August 3–4, 2023
New York City

Thursday, August 3

9:15 - 9:30	Opening remarks
9:30 - 10:30	**Anthony Bonato**, Toronto Metropolitan University *Progress on Pursuit-Evasion Games on Graphs*
10:30 - 10:45	BREAK
10:45 - 11:15	**John Marcoux**, Toronto Metropolitan University *k-Visibility Localization (Part 1)*
11:15 - 11:45	**Trent Marbach**, Toronto Metropolitan University *k-Visibility Localization (Part 2)*
11:45 - 1:30	LUNCH
1:30 - 2:00	**Jessica Enright**, University of Glasgow *Multilayer cops-and-robbers*
2:00 till Dinner Time!	OPEN DISCUSSION

Friday, August 4

9:30 - 10:30	**Abigail Raz**, Cooper Union *The Explorer-Director Game on Graphs*
10:30 - 10:45	COFFEE
10:45 - 11:15	**Danny Dyer**, Memorial University of Newfoundland *An introduction to cat herding*
11:15 - 11:45	**Niko Townsend**, University of Rhode Island *High-Speed Cops and Robbers*
11:45 - 1:30	LUNCH
1:30 - 2:00	**Evan MacTavish**, University of New Brunswick *The donut-shop model of Cops and Robbers*
2:00 till Dinner Time!	OPEN DISCUSSION

Abstracts of Talks

Anthony Bonato, Toronto Metropolitan University — Thursday
Progress on Pursuit-Evasion Games on Graphs — 9:30 - 10:30

We overview recent work on pursuit-evasion games on graphs, including Cops and Robbers, burning, and the Localization game.

John Marcoux, Toronto Metropolitan University — Thursday
k-Visibility Localization (Part 1) — 10:45 - 11:15

A common way of creating interesting variants of pursuit-evasion games is to restrict the information that the pursuers have. In this talk we explore one such variant known as *Limited Visibility Localization* or *k-Visibility Localization*, a version of localization where a cop's probe only works if the robber is within distance k of said cop. The talk will introduce the concept of *k-Visibility Localization* and give some general intuition for how the game behaves in contrast to standard localization. The talk will also cover results about subdivisions of trees which reduce the required number of cops to 1 and why this is not always possible when $k = 1$.

This is a two part talk with Trent Marbach and is based on joint work with Anthony Bonato, Trent Marbach, and JD Nir.

Trent Marbach, Toronto Metropolitan University — Thursday
k-Visibility Localization (Part 2) — 11:15 - 11:45

For many pursuit-evasion games on graphs, it is possible to consider a k-visibility variant, where the evader is invisible unless some pursuer has distance at most k from the evader. The pursuers' strategy in these variant games can naturally be broken down into two phases, where the pursuers first aim to 'see' the evader and then try to capture the evader. When k is small, these variant games are often similar to each other, and have strong connections to the isoperimetric parameters of graphs. When k is larger, the variant games are not very different to the original games.

In this presentation, we focus on the localization game, in which during their turn, the pursuers know the distance from each pursuer to the evader. We study the k-visibility localization game played on square grid graphs and show that a lower bound on the number of pursuers required can be found using the isoperimetric parameters of these graphs and, quite surprisingly, this lower bound is very close to being tight. In apparent contrast to this, we also provide results of this game on trees, in which the isoperimetric-based lower bounds do not seem to perform well and so introduces some interesting complications.

This is a two part talk with John Marcoux and is based on joint work with Anthony Bonato, John Marcoux, and JD Nir.

Jessica Enright, University of Glasgow — Thursday
Multilayer cops-and-robbers — 1:30 - 2:00

In a multilayer setting, a graph consists of a single set of vertices with multiple (potentially intersecting) edge sets. We allow the cops and robber to move only on their assigned layer, and ask if the cops can be guaranteed to catch the robber in finite time. Using several examples, I'll show that initial intuition about the best way to allocate cops to layers is not always correct. I will outline arguments showing that the number of cops required to catch a robber in a multilayer graph is neither bounded from above nor below by any function of the cop numbers of the individual layers. Additionally, we'll talk about a question of worst-case division of a simple graph into layers: given a simple graph G, what is the maximum number of cops required to catch a robber over all multilayer graphs where each edge of G is in at least one layer and all layers are connected? For cliques, suitably dense random graphs, and graphs of bounded treewidth, we have determined this parameter up to multiplicative constants.

Abigail Raz, Cooper Union — Friday
The Explorer-Director Game on Graphs — 9:30-10:30

Abstract: The Explorer-Director game, first introduced by Nedev and Muthukrishnan, can be described as a game where two players—Explorer and Director—determine the movement of a token that is positioned on the vertices of some given graph. At each time step, the Explorer specifies a distance that the token must move with an aim to maximize the total number of vertices ultimately visited. However, the Director adversarially chooses where to move token in an effort to minimize this number. The game ends when no new vertices can be visited. Given a graph G and a starting vertex v, the number of vertices that are visited under optimal play is denoted by $f_d(G, v)$. In this talk we will explore this game providing some general bounds, exact results on $f_d(G, v)$ for specific graph families, and, time permitting, discuss some scenarios when the Explorer can still force $f_d(G, v)$ vertices to be visited with a non-adaptive strategy (i.e. the Explorer's choices are independent of the Director's choices).

This research is joint with Pat Devlin, Erin Meger, and a group of over 20 undergraduate students participating in the 2020 Polymath REU.

Danny Dyer, Memorial University of Newfoundland — Friday
An introduction to cat herding — 10:45 - 11:15

Cat herding is a pursuit-evasion game where a fast, visible cat attempts to elude being captured by a herder for as long as possible. The herder attempts to isolate the cat on a vertex by deleting edges, and the minimum number of edges needed to isolate the cat is the cat number. After introducing this model, we will discuss the cat number of some classic families of graphs, and the difficulties faced in attempted to capture a cat in trees and complete graphs.

Niko Townsend, University of Rhode Island — Friday 11:15 - 11:45
High-Speed Cops and Robbers

In recent years, there has been considerable interest in variants of Cops and Robbers wherein the robber moves "faster" than the cops. What if we increased the cops' speed, as well? In general, a cop or robber can be assigned a speed s, meaning that they are able to move along a path of length at most s on each turn. We consider the variant of the game in which each of the cops and the robber are given speed s, with $s > 1$. In this talk, we will present an overview of our work on this variant, with a focus on similarities to and differences from the original game. We will also pose some tantalizing open questions. This is joint work with William B. Kinnersley.

Evan MacTavish, University of New Brunswick — Friday 1:30 - 2:00
The donut-shop model of Cops and Robbers

We introduce a variation of the game of Cops and Robbers on graphs, in which cops pursue a robber in the presence of one or more special vertices: donut-shops. The location of a donut-shop is determined prior to each player selecting their starting positions, and should a cop ever move to a donut-shop, they are unable to move for the remainder of the game. These conditions form the donut-shop model for Cops and Robbers, which introduces a new parameter: the donut-shop cop-number. We will look at some initial results depending on the vertex that the donut-shop is established on, and we will consider the donut-shop model on various classes of graphs. This is joint work with Andrea Burgess and Alyssa Sankey.