ONTARIO COMBINATORICS WORKSHOP

May 6–7, 2011
Ryerson University
Toronto, Canada

Friday, May 6

8:30 - 9	Registration and opening remarks
9:00 - 10	Jeannette Janssen **Plenary lecture** *Spatial models for virtual networks*
10 - 10:30	COFFEE
10:30 - 11	Maryam Haghighi *A graph-theoretical approach to constructing evolutionary trees*
11 - 11:30	Yuval Filmus *Triangle-intersecting families of graphs*
11:30 - 12	Amanda Tian *Models and mining of on-line social networks*
12:00 - 1	LUNCH
1 - 1:30	Liam Baird *Meyniel extremal families of graphs*
1:30 - 2	Abbas Mehrabian *Cops and Robber game with a fast robber on expander graphs and random graphs*
2:00 - 3	Jacques Verstraete **Plenary lecture** *The Turán problem: theory and applications*
3 - 3:30	COFFEE
3:30 - 4	Tamar Krikorian *Relationships between OAs, OOAs and* (t, m, s)*-nets*
4 - 4:30	Beth Ann Austin *Categorizing some 2-crossing critical-graphs*
4:30 - 5	Andrew McConvey *Crossing sequences of graphs*
5 - 5:30	Lei Xu *Computing the map of geometric minimal cuts*
5:30 - 6	Richard Hoshino *Optimizing the scheduling of Nippon Professional Baseball using graph theory*

Saturday, May 7

9:00 - 10	Brett Stevens **Plenary lecture** *Optimizing an imperfect tournament*
10 - 10:30	COFFEE
10:30 - 11	Andrew Skelton *Response curves in deterministic and probabilistic cellular automata*
11 - 11:30	Huda Chuangpishit *Nowhere-zero flows of graphs*
11:30 - 12	Andrew MacFie *Counting words by number of occurrences of a pattern*
12:00 - 1	LUNCH
1 - 1:30	Craig Sloss *A character-based solution to a special case of the* (p, q, n)*-dipole problem*
1:30 - 2	Emily Redelmeier *Fluctuations of real random matrices*
2 - 2:30	Robert Bailey *A matrix method for resolving sets in Johnson graphs*
2:30 - 3	COFFEE
3 - 3:30	Robin Christian *Cycle spaces of graph-like continua*
3:30 - 4	Nevena Francetic *Covering arrays with row limit* $w = 4$
4 - 4:30	Andrea Burgess *The Erdős-Lovász Tihany Conjecture and block colourings of Steiner triple systems*
4:30 - 5	Frederic Edoukou *Weight distribution of the Hermitian code defined on a non-singular surface*
5 - 5:30	Pawel Pralat *Meyniel's conjecture holds for random graphs*
5:30 - 6	Graeme Kemkes *Almost all cop-win graphs have a universal vertex*
6 - 6:15	Peter Rodney Memorial Book Prize and closing remarks

Plenary speakers

Jeannette Janssen, Dalhousie University — Friday
Spatial models for virtual networks — 9:00 - 10

In an attempt to explain the link structure of complex networks such as web graphs, on-line social networks, or citation graphs, increasing emphasis is being placed on spatial graph models. In a spatial graph model, nodes are embedded in a metric space, and link formation depends on the relative position of nodes in the space. Spatial models form a good basis for link mining: assuming a spatial model, the link information can be used to infer the spatial position of the nodes, and this information can then be used for clustering and recognition of node similarity. I will discuss recent results on spatial graph models, including joint work with Anthony Bonato and Pawel Prałat.

Brett Stevens, Carleton University — Saturday
Optimizing an imperfect tournament — 9:00 - 10

A computer science department holds an annual video game olympics with 64 participants playing 8 games. There are 8 rooms each with a fixed video game and there are 8 rounds. In each round 8 people will be in each room. Every person will play each game exactly once. We would like to find a schedule for all the players, rooms and rounds that is as balanced as possible, i.e. no pair of players plays together in the same room too frequently and as few pairs of people playing together are missed. It can be shown that some pairs must be missed and some pairs must repeat. We set up a combinatorial framework to quantify the repetition and missing pairs and try several different approaches to optimise the tournament. For various criteria we will find solutions constructed from lines in finite planes, ovals in finite geometries and finally a set of solutions that are related to Costas sonar and radar sequences.

Jacques Verstraete, University of California, San Diego — Friday
The Turán problem: theory and applications — 2:00 - 3

For graphs F and G, we say that G is F-free if G contains no isomorphic copy of F as a subgraph. The *Turán number* for F, denoted $\mathrm{ex}(n, F)$, is the maximum number of edges in an n-vertex F-free graph. The study of this quantity is a cornerstone of extremal combinatorics, and has connections to many other areas of mathematics and theoretical computer science. In this talk, I shall present some of these remarkable connections, including applications to number theory, combinatorial geometry, group theory, information and coding theory, and randomized algorithms.

Contributed talks

Beth Ann Austin, University of Waterloo — Friday
Categorizing some 2-crossing critical-graphs — 4 - 4:30

The crossing number of a graph is the minimum number of edges crossed among all drawings of the graph in the plane. Of particular interest are crossing-critical graphs: graphs for which any subgraph has a smaller crossing number than the original graph.

A great deal of work has been done to characterize all 2-crossing-critcal graphs. Our research is working towards filling the gap of 3-connected, 2-crossing-critical graphs that contain a V_8 minor but no V_{10}. A substantial list of such graphs exist and the structure of the V_8 offers tools to find others. The classification of these graphs would complete the determination of all 2-crossing-critical graphs, as those that are not 3-connected are either known or easily obtained from those that are 3-connected.

Robert Bailey, University of Regina — Saturday
A matrix method for resolving sets in Johnson graphs — 2 - 2:30

A *resolving set* for a graph G is a subset of vertices with the property that the list of distances from a vertex to the chosen set uniquely identifies that vertex. The *metric dimension* of G is the smallest size of a resolving set.

In this talk, we use incidence matrices of set systems to obtain resolving sets for the Johnson graph $J(n,k)$, and show how some interesting combinatorial objects, such as projective planes and Hadamard matrices, can be used to construct resolving sets.

Liam Baird, Ryerson University — Friday
Meyniel extremal families of graphs — 1 - 1:30

Cops and Robbers is a vertex-pursuit game played on a graph, where a set of cops attempts to capture the robber. The minimum number of cops needed to win is the *cop-number* of the graph G, written $c(G)$. Perhaps the deepest open problem in the study of Cops and Robbers is Meyniel's Conjecture. The conjecture is that for a connected graph G, $c(G) = O(\sqrt{|V(G)|})$.

A Meyniel extremal family of graphs G_n where $|V(G_n)| = n$, satisfy $c(G_n) \geq d\sqrt{n}$ for some constant d. One such Meyniel extremal family arises as the incidence graphs of projective planes. We give new examples of Meyniel extremal families.

Andrea Burgess, Ryerson University Saturday
The Erdős-Lovász Tihany Conjecture and block colourings of Steiner triple systems 4 - 4:30

The Erdős-Lovász Tihany Conjecture posits that for any graph G with $\chi(G) > \omega(G)$, and any integers $s, t \geq 2$ with $s + t = \chi(G) + 1$, the vertices of G can be partitioned into two sets, S and T, such that the subgraphs of G induced by S and T have chromatic numbers $\chi(G[S]) \geq s$ and $\chi(G[T]) \geq t$. This conjecture has been proved for certain classes of graphs, including line graphs and quasi-line graphs. We consider the problem of determining whether the conjecture holds for block intersection graphs of Steiner triple systems, noting that vertex colourings of such graphs correspond with block colourings of the associated triple systems, and discuss progress we have made towards its solution in certain cases.

Robin Christian, University of Waterloo Saturday
Cycle spaces of graph-like continua 3 - 3:30

A graph-like space is a topological space that resembles a graph, in the sense that it is the union of edges (which are homeomorphs of the unit interval) and vertices (which form a totally disconnected subset of the space). A continuum is a topological space that is compact, connected and Hausdorff.

We show that, for any edge e of a 3-connected graph-like continuum, there are two peripheral circuits, each of which contain e and which are otherwise disjoint. The specialization of this result to 3-connected finite graphs is a classical result of Tutte. Time permitting, we discuss the generalization of another of Tutte's results: the peripheral circuits of a 3-connected graph-like continuum generate its cycle space.

This is joint work with Bruce Richter.

Huda Chuangpishit, Dalhousie University Saturday
Nowhere-zero flows of graphs 11 - 11:30

A graph G has a nowhere-zero k-flow if the edges of G can be oriented and assigned numbers $\pm 1, \ldots, \pm(k-1)$ such that for every vertex, the sum of the values on incoming edges equals the sum on the outgoing ones. This concept was introduced by Tutte in 1945. He showed that each planar graph is k-face colorable if and only if it has a nowhere-zero k-flow. There are three celebrated conjectures in this field, all due to Tutte. In sequel Jeager et al. introduced the concept of group connectivity of graphs as an extension of nowhere-zero flows. We will discuss the most important results on nowhere-zero flows and the concept of group connectivity.

Frederic Edoukou, Nanyang Technological University, Singapore — Saturday
Weight distribution of the Hermitian code defined on a non-singular surface — 4:30 - 5

Hermitian codes defined by evaluating quadratic functions on the non-singular Hermitian surface over the field of four elements have been studied by several authors such as P. Spurr, R. Tobias and I. M. Chakravarti with the help of computer programs.

In this talk without any computer programs, we will generalize all the results obtained by the three authors, by tools coming from algebraic geometry, finite geometry and combinatorics. We will find the minimum distance, the weight distribution and the divisor of the Hermitian code for any arbitrary field.

This is a joint work with San Ling (NTU, Singapore) and Chaoping Xing (NTU, Singapore).

Yuval Filmus, University of Toronto — Friday
Triangle-intersecting families of graphs — 11 - 11:30

A family of graphs (sharing the same finite vertex set) is *triangle-intersecting* if the intersection of any two graphs in the family contains a triangle. Simonovits and Sós conjectured in 1976 that such a family can contain at most 1/8 of the graphs; this bound is achieved by taking a "triangle-junta" (all graphs containing a fixed triangle).

Up to the present work, the best known upper bound on the size of a triangle-intersecting family was 1/4 (Chung, Graham, Frankl & Shearer 1981). We prove the conjecture, showing moreover that the only families achieving the bound are triangle-juntas.

The result follows a line of work by Friedgut and his coauthors. The proof uses rudimentary discrete Fourier analysis, and some "entry-level" graph theory; the talk will focus on the former.

Joint work with David Ellis (Cambridge) and Ehud Friedgut (Hebrew University)

Nevena Francetic, University of Toronto — Saturday
Covering arrays with row limit $w = 4$ — 3:30 - 4

The Covering Arrays with Row Limit ($CARL$) is a generalization of a covering array. It is a combinatorial model of a test suite, which allows us to test only a subset of the components in each test run. The parameter w, which corresponds to the number of components tested at a time, is called **weight**. Even though $CARL$s are newly introduced combinatorial objects, they are closely related to the well studied group divisible designs, covering arrays, and graph coverings. For a fixed value of w, we present the lower bound on the size of a $CARL$. Also, we present some constructions of optimal $CARL$s with $w = 4$, strength $t = 2$, and a regular excess graph. Further research is needed to find the examples of the optimal auxiliary ingredient $CARL$s with irregular excess graphs, used in these constructions to provide an optimal solution for any $CARL$ with $w = 4$ and $t = 2$.

Maryam Haghighi, University of Ottawa — Friday 10:30 - 11
A graph-theoretical approach to constructing evolutionary trees

In recent years, there has been significant interest on the connections between Combinatorics and Computational Biology. With the large amount of genomic data becoming available, it is now possible to compare different genomes. This is particularly important since when developing drugs, we typically test them on mice before human. So a fundamental question is how much evolution separates different species from each other.

We show how to model this problem using graphs. We study attributes of these graphs and use them to obtain a novel approximation algorithm for the NP-hard problem of constructing evolutionary trees. This graph-theoretical approach proves to be very powerful especially considering the size of genomes. In this talk, we present new theoretical results which provide improvements to the solution, followed by results from an empirical study.

Richard Hoshino, National Institute of Informatics, Tokyo — Friday 5:30 - 6
Optimizing the scheduling of Nippon Professional Baseball using graph theory

In this talk, we consider the role of mathematically-optimal schedules for professional sports leagues, specifically for the popular Nippon Professional Baseball (NPB) League in Japan. We tackle this challenge by generalizing the Traveling Tournament Problem (TTP), whose output is a double round-robin schedule that minimizes the sum total of distances traveled by the n teams as they move from city to city, subject to several natural constraints to ensure balance and fairness.

To optimize the NPB *intra-league* schedule, we develop a multi-round generalization of the TTP and present an algorithm that converts the problem to finding the shortest path in a directed graph. To optimize the NPB *inter-league* schedule, we develop a bipartite generalization of the TTP, and apply a heuristic based on minimum-weight cycle covers.

Compared to the actual distance traveled by these teams during the 2010 NPB season, our provably-optimal schedules reduce the total travel by 67,000 kilometres!

Graeme Kemkes, Ryerson University — Saturday 5:30 - 6
Almost all cop-win graphs have a universal vertex

A cop and a robber take turns moving from vertex to vertex on a graph. The graph is *cop-win* if the cop has a strategy to land on the same vertex as the robber eventually. We show that the proportion of cop-win graphs having a universal vertex (i.e., a vertex adjacent to all other vertices) tends to 1 as the number of vertices grows large. Joint work with Anthony Bonato and Paweł Prałat.

Tamar Krikorian, Ryerson University — Friday 3:30 - 4

Relationships between OAs, OOAs and (t, m, s)-nets

Low discrepancy point sets are used in quasi-Monte Carlo integration to approximate the value of an integral of multiple dimensions. One of the most effective known constructions for low discrepancy point sets in the s-dimensional unit cube is based on the theory of (t, m, s)-nets. (t, m, s)-nets are equivalent to a parametric subclass of a combinatorial object called ordered orthogonal arrays (OOA). Known constructions for these OOA's involve methods from areas such as coding theory and finite projective geometry. In this talk, we present new methods of construction for OOA's with certain properties, and we also present new theorems involving graph homomorphisms that relate OOA's to orthogonal arrays (OA). We also introduce a new combinatorial object called ordered covering arrays, and discuss how they can be used to construct (t, m, s)-nets.

Andrew MacFie, Carleton University — Saturday 11:30 - 12

Counting words by number of occurrences of a pattern

Given words σ and τ over an ordered finite alphabet, where τ is called the pattern, an occurrence of τ in σ is a subsequence of σ that is order-isomorphic to τ — that is, a subsequence of σ equal in length to τ such that the relative order of two elements in the subsequence is the same as the relative order of the corresponding elements of τ. For example, all length 3 subsequences of $\sigma = 1234$ are order-isomorphic to $\tau = 123$.

Not much is known about the number of words of length n from an alphabet of size k with r occurrences of a pattern for general r. I will present a recurrence followed by an asymptotic expression for this when the pattern is $\tau = 12...2$ which shows a connection to a type of integer partitions.

Andrew McConvey, University of Waterloo — Friday 4:30 - 5

Crossing sequences of graphs

For a given graph, G, the crossing number $\mathrm{cr}_k(G)$ denotes the minimum number of edge crossings when a graph is drawn on a surface of genus k. The sequence $\mathrm{cr}_0(G),\ \mathrm{cr}_1(G), ...$ is said to be the crossing sequence of a G.

In 1983, Jozef Širáň proved that for every decreasing, convex sequence of non-negative integers, there is a graph G such that this sequence is the crossing sequence of G. In 2001, Archdeacon, Bonnington, and Širáň showed that there exists a graph with arbitrary crossing sequence $a, b, 0$. Despite recent progress, it remains an open question as to whether there can exist a graph with a non-convex crossing sequence of arbitrary length. This talk will provide an overview of crossing sequences, recent results, and directions of current research.

Abbas Mehrabian, University of Waterloo — Friday 1:30 - 2
Cops and Robber game with a fast robber on expander graphs and random graphs

We consider a variant of the Cops and Robber game in which the robber has unbounded speed. The minimum number of cops needed to capture the robber in a graph G is called the cop number of G. We show lower bounds for cop number in terms of vertex and edge expansion of the graph. We apply these results to random graphs to prove that

1. For any fixed d, the random d-regular graph on n vertices a.a.s has cop number $\Theta(n)$.
2. For $np \gg \log n$, the random graph $G(n,p)$ a.a.s has cop number $\Omega(1/p)$.

Pawel Pralat, Ryerson University — Saturday 5 - 5:30
Meyniel's conjecture holds for random graphs

We study the vertex pursuit game of *Cops and Robbers*, in which cops try to capture a robber on the vertices of the graph. The minimum number of cops required to win on a given graph G is called the cop number of G. We present asymptotic results for the game of Cops and Robber played on a random d-regular graph $G_{n,d}$ and a binomial random graph $G(n,p)$. In particular, we show that the Meyniel's conjecture holds asymptotically almost surely for $G_{n,d}$ with $d \geq 3$ as well as for $G(n,p)$ model with $p = p(n)$ above the threshold for connectivity. (Joint work with Nick Wormald.)

Emily Redelmeier, Queen's University — Saturday 1:30 - 2
Fluctuations of real random matrices

We will discuss the combinatorial constructions used to calculate the global fluctuations of real random matrices, in particular unoriented surface gluings (as opposed to the oriented ones appearing in the complex case) and the cartographic machinery used in this context. We will examine the large-matrix behaviour exhibited by real random matrices and the types of noncrossing diagrams which may be used to calculate the fluctuations of products of independent matrix ensembles.

Andrew Skelton, Brock University — Saturday 10:30 - 11
Response curves in deterministic and probabilistic cellular automata

Cellular automata (CA) have been used as discrete models in areas as diverse as disease spread, forest fires and traffic flow. One of the most important properties in any binary CA model is the proportion of cells in a specific state (0 or 1) after a given number of time iterations. We approach this problem using patterns in preimage sets - that is, the set of blocks which iterate to our desired output. This allows us to construct a response curve - a relationship between the proportion of cells in state 1 after n-iterations as a function of the initial proportion of cells in state 1. We first derive response curves for a class of two-dimensional deterministic CA rules, including an important subset of surjective rules. We conclude by considering a class of one-dimensional probabilistic CA rules.

Craig Sloss, University of Waterloo — Saturday 1 - 1:30
A character-based solution to a special case of the (p, q, n)*-dipole problem*

Enumerative questions about factorizations in the symmetric group have applications to areas including algebraic geometry, topology, physics and computer science. Character theory of the symmetric group is an important method for studying these problems, but it can only be used to study problems for which the answer is a function of the conjugacy classes of the permutations involved. Such problems are called central problems. The (p, q, n)-dipole problem, which arises in theoretical physics, is an example of a permutation factorization problem which is non-central. A combinatorial argument is used to show that an important special case ($q = n - 1$) is "close enough" to being central to permit it to be studied using a refinement of character-based methods. This leads to a solution to the $(p, n - 1, n)$-dipole problem on all orientable surfaces, and provides a method to study non-central analogues of other combinatorial problems.

Yanhua (Amanda) Tian, Ryerson University — Friday 11:30 - 12
Models and mining of on-line social networks

Power law degree distribution, small world property, and bad spectral expansion are three of the most important properties of On-line Social Networks (OSNs). We use real OSNs samples to calculate and verify those three properties. We introduce the GEO-P models which naturally simulate OSNs. We use the GEO-P models to simulate OSNs, and verify how they fit OSNs by calculating the three main properties. The calculation results are compared to real OSNs to support the Logarithmic Dimension Hypothesis, which conjecture the dimension of an OSN is $m = \lceil \log N \rceil$, where N is the number of nodes in the network.

Lei Xu, State University of New York at Buffalo — Friday 5 - 5:30
Computing the map of geometric minimal cuts

In this talk we consider the problem of computing a map of geometric minimal cuts (called MGMC problem) induced by a planar rectilinear embedding of a subgraph $H = (V_H, E_H)$ of an input graph G. We first show that unlike the classic min-cut problem on graphs, the number of all rectilinear geometric minimal cuts is bounded by a low polynomial, $O(n^3)$. Our algorithm for identifying geometric minimum cuts runs in $O(n^3 \log n (\log\log n)^3)$ time in the worst case which can be reduced to $O(n \log n (\log\log n)^3)$ when the maximum size of the cut is bounded by a constant, where $n = |V_H|$. Once geometric minimal cuts are identified we show that the problem can be reduced to computing the L_∞ Hausdorff Voronoi diagram of axis aligned rectangles. We present the first output-sensitive algorithm to compute this diagram which runs in $O((N + K) \log^2 N \log\log N)$ time and $O(N \log^2 N)$ space, where N is the number of rectangles and K is the complexity of the diagram. Finally we extend our result to the case in which the embedding graph contains 45-degree edges.

FIELDS

This workshop is generously funded by the Fields Institute for Research in Mathematical Sciences.